Alternating Signal Temporal Logic

IFIP WG1.3

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An IFIP TC1 Working Group

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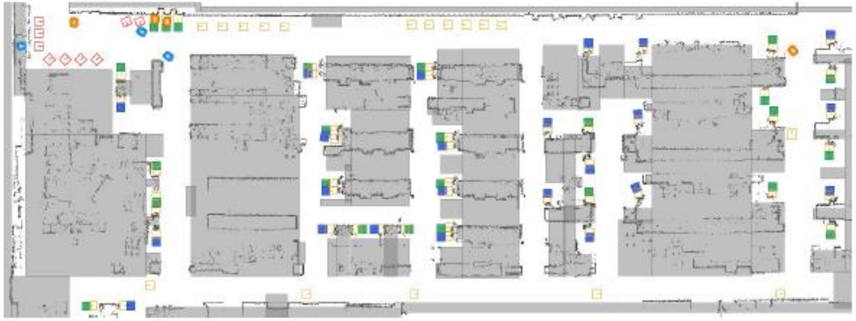
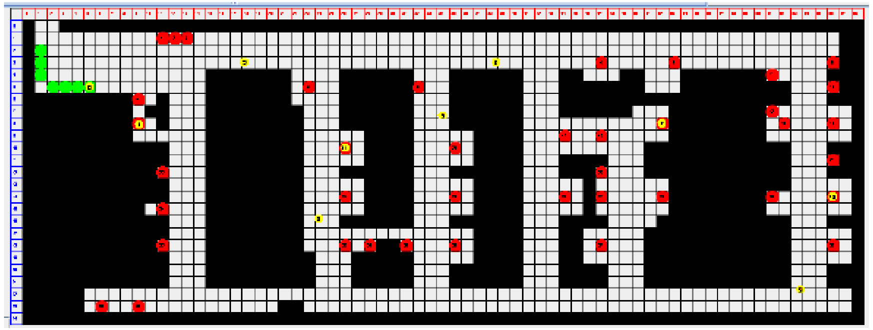
Motivation

Foundations of Systems Specification

- Formal methods in real applications
- Scheduling of transport robots
- Controller design
- Specification and verification of industrial systems





Structure of this talk

- More motivation
- Concurrent game structures and strategic logics
- Hybrid automata and signal temporal logics
- Combining strategic and continuous systems and logics
- Model checking results
- Perspectives and outlook

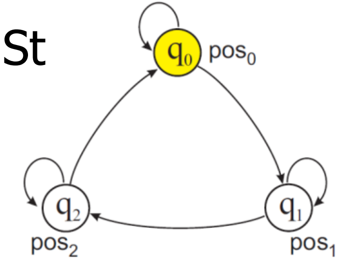
Industrial Multi-Agent Systems

- Real-time / hybrid
  - Continuous sensor inputs, discrete control, continuous motor outputs
- Inherently distributed, space might be important
  - Unreliable communication, intrusion
- "Intelligent" and autonomous
  - Beliefs, intentions, desires; fuzzy goals

- Research questions
  - How to model such systems?
  - How to specify properties?
  - How to synthesize winning strategies?

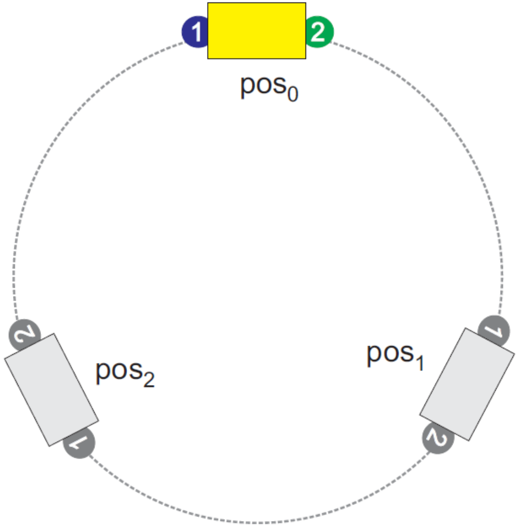
Concurrent Game Structures

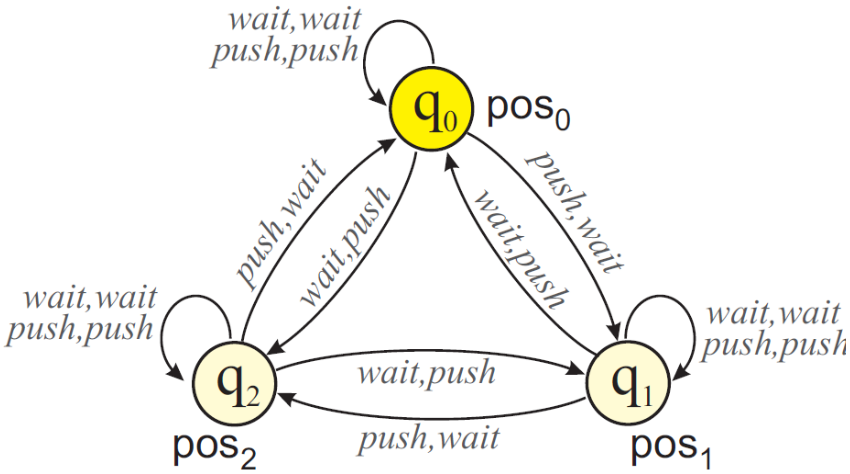
- Classical definitions
  - Transition system TS = (St, δ, s_0), $\delta \subseteq St \times St$, $s_0 \in St$
  - Labelled transition system LTS = (St, Act, δ, s_0), $\delta \subseteq St \times Act \times St$
  - Kripke structure KS = (St, Prop, δ, Int, s_0), $Int \subseteq St \times Prop$
  - Finite state machine, Büchi/Rabin/Muller automaton, …



- Several LTS's: ($LTS_1 \times \ldots \times LTS_n$)
  - Product transition system – synchronization via shared actions
  - Concurrent game structure CGS = (Agt, St, Act, δ, s_0), $\delta \subseteq St \times 2^{(Agt \times Act)} \times St$
    combined action $\alpha = \{(a_1, \alpha_{a1}), \ldots, (a_n, \alpha_{an})\}$, every agent at most one action
    ("agent a_1 chooses α_{a1} and … and agent a_n chooses α_{an}")

Example: Robots and Carriage





Thanks to Wojtek Jamroga and Wojtek Penczek

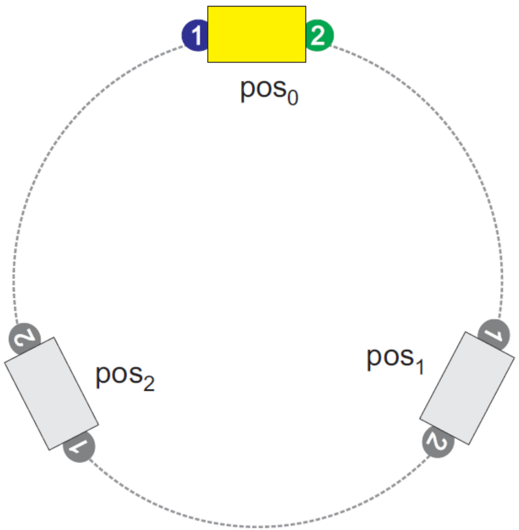
- Variants
  - Deterministic CGS with availability relation DCGS = (Agt, St, Act, *avail*, δ, s_0), *avail* $\subseteq$ Agt x St x Act, δ: St x $2^{(Agt \times Act)} \rightarrow$ St is a function, (s, α , s') $\in$ δ only if (a, s, α_a) $\in$ *avail*
  - Synchronous game structure: all agents have to choose an action in each state (that is, δ: St x $Act^n \rightarrow$ St)
  - Coalitional game structure: agents can form a coalition and choose their actions synchronously
  - Turn-based game structure: agents or coalitions take turn with their actions; TCDCGS = (Agt, St, Act, *avail*, δ, *turn*, s_0), *turn*: St $\rightarrow$ Agt; if *turn* (s) = a_i, then *avail* (a_i, s) $\neq \emptyset$, and for all j≠i, *avail* (a_j, s) = $\emptyset$.

Goals and Strategies

- In the simple case, a goal is a designated set of states
  - more advanced goals can be described by logic, cf. later
- Strategy $\mathbf{s}_i$ for a_i in a DCGS $\mathbf{s}_i$: St → Act s.t. $(a_i, s, \mathbf{s}_i(s)) \in avail$
  - in case $\{\alpha_a \mid (a_i, s, \alpha_a) \in avail\} = \emptyset$, there is no strategy for a_i
- Combined action α is consistent with the strategies $\{\mathbf{s}_1 \ldots \mathbf{s}_k\}$ for agents $\{a_1 \ldots a_k\}$ in state s if $(a_i, \mathbf{s}_i(s)) \in \alpha$ for all i
- An execution σ following strategies $\{\mathbf{s}_1, \ldots, \mathbf{s}_k\}$ for agents $\{a_1, \ldots, a_k\}$ is an execution $(s_0, \alpha_0, s_1, \alpha_1, \ldots)$, where every α_i is consistent with the strategies in s_i
  - Given strategies for all agents in a DCGS, there is only one possible execution following all these strategies (usually called the outcome)

Example continued

- In the initial state, does Robot_1 have a strategy to bring the carriage into position pos_1?
- Do Robot_1 and Robot_2 have a combined strategy to reach any desired position?
- Does Robot_1 have a strategy to <u>avoid</u> pos_1?



Alternating Temporal Logic

- Syntax: $\langle\langle A \rangle\rangle \varphi$
  where A is a set of agents and φ is an LTL formula
- Semantics: $\mathcal{M} \models \langle\langle A \rangle\rangle \varphi$ iff for each a_i in A there is a strategy $\mathbf{s_i}$ such that in each execution σ of **M** which follows all these $\mathbf{s_i}$, it holds that $\sigma \models \varphi$
  - Note that $\langle\langle a_1, a_2 \rangle\rangle\ \varphi$ is not the same as $\langle\langle a_1 \rangle\rangle\ \langle\langle a_2 \rangle\rangle\ \varphi$!

- Examples
  - $\langle\langle Robot_1 \rangle\rangle\, \mathbf{F}\, pos_1$ – $Robot_1$ has a strategy to get to pos_1
  - $\langle\langle \{Robot_1, Robot_2\} \rangle\rangle\, \mathbf{F}\, pos_1$ – A combined strategy to reach pos_1
  - $\langle\langle Robot_1 \rangle\rangle\, \mathbf{G}\, \neg pos_1$ – A strategy to avoid pos_1

ATL Complexity and Algorithm

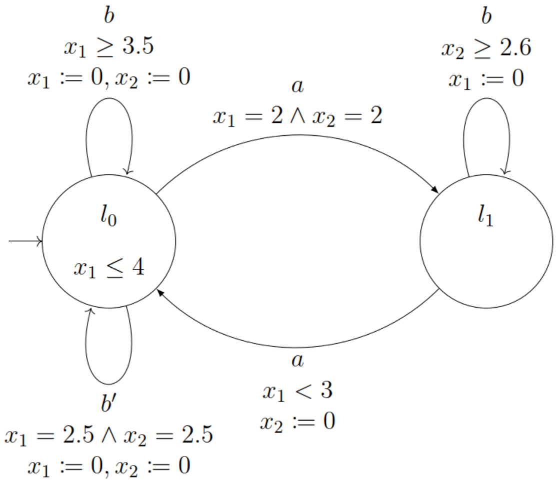
- Variants
  - ATL: every temporal operator preceded by exactly one cooperation modality
  - ATL*: no syntactic restriction
- Complexity results
  - ATL on CGS is in **P** (fixpoint unwinding)
  - ATL on CEGS with memory is undecidable

	Ir	IR	ir	iR
Simple $\mathscr{L}_{CL}$	Σ_2^P	Σ_2^P	Σ_2^P	Σ_2^P
$\mathscr{L}_{CL}$	Δ_3^P	Δ_3^P	Δ_3^P	Δ_3^P
$\mathscr{L}_{ATL}$	Δ_3^P	Δ_3^P	Δ_3^P	Undecidable†
$\mathscr{L}_{ATL^+}$	Δ_3^P	*PSPACE*	Δ_3^P	Undecidable†
$\mathscr{L}_{ATL^*}$	*PSPACE*	*2EXPTIME*	*PSPACE*	Undecidable†

- Algorithms
  - Calta, Schlingloff (2010): $O(l*n^2*3^{(2*n*a^k/3)})$-algorithm for CEGSs (l strategic modalities, n states, k agents, a actions)
  - Lomuscio et al (2015): MCMAS tool for ATLK on CGS (ISPL)

Timed and Hybrid Automata

- Basically, a HA is an LTS with additional real-valued variables
  - Finite set of *locations* - a *state* is a location plus a variable valuation
  - ➔ infinitely many states!
  - A *region* is a (convex) set of states
  - Locations and/or transitions may be constrained by equations using variables
  - Variables and their derivatives may be assigned values



- Timed automata (TA)
  - A clock is a variable where the derivative is always 1
  - All clocks always advance with the same speed, no stopwatches

Signal Temporal Logic

- Syntax and semantics borrowed from interval temporal logic
- Basic propositions: (s ~ c)
  - s is a real-valued signal (i.e., variable or clock), c is a constant
- Boolean junctors, interval temporal until:
  - $(S,t) \models (\varphi_1 \, \mathbf{U}_I \, \varphi_2) \longleftrightarrow \exists t_1 \in t + I \; (S,t_1) \models \varphi_2 \text{ and } \forall t_2 \in [t,t_1) \; (S,t_2) \models \varphi_1$
- Examples
  - $\mathbf{G}_{(0,\infty)}(sensor.USfront \leq 5)$ – (no crash)
  - $\mathbf{F}_{(0,30]}((clock \geq 300) \vee (collect = true)))$ – (finding items in time)
  - $\mathbf{G}_{[0,\infty)}((sensor.r \geq 200) \rightarrow ((wheels = -5) \, \mathbf{U}_{(2,4)} \, (sensor.r \leq 200)))$ – (how to drive)

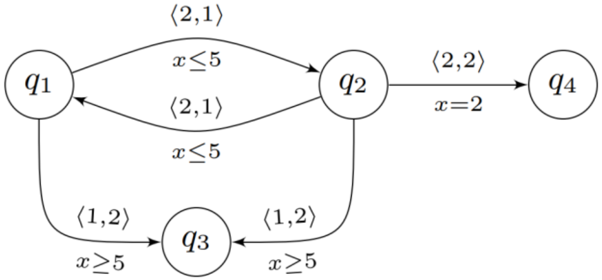
Model Checking STL on HA

- Reachability for hybrid automata is undecidable
  - but can be easily expressed in the considered logics
  - ➔ no hope to come up with a terminating model checking algorithm
- Reachability for timed automata is decidable (PSPACE-complete)
  - model checking for STL on TA can be reduced to this problem
  - Region-Graph and Difference-Bound Matrix construction of [Yovine 97, Alur 98]
- “Rectangular” hybrid automata
  - first derivative is bounded by constants
  - LTL model checking on RHA is in PSPACE

Combining Continuity and Strategies

Timed and hybrid CGS

- A fusion of CGS and timed / hybrid automata
- Formally, an automaton with dedicated actions for each player
  - Transitions are labelled with concurrent actions and activation conditions on the clocks or continuous variables
- Additional requirements: determinism, non-Zeno-ness
- Given a choice of strategies for all players, there is at most one run of the automaton



http://www.lsv.fr/Projects/anr-dots/PUBLIS/BLMO-concur07.pdf

Alternating Signal Temporal Logic

- Distinguish between monitored and controlled variables (sensors and actuators)
- Strategic decisions concern controlled variables

$$
\begin{aligned}
(\mathcal{T}, Q_0) &\models \top && \longleftrightarrow \forall l \in L\; Q_0^l \models I(l) \\
(\mathcal{T}, Q_0) &\models x \sim c && \longleftrightarrow Q_0 \models x \sim c \text{ and } \forall l \in L\; Q_0^l \models I(l) \\
(\mathcal{T}, Q_0) &\models \neg\varphi && \longleftrightarrow (\mathcal{T}, Q_0) \not\models \varphi \text{ and } \forall l \in L\; Q_0^l \models I(l) \\
(\mathcal{T}, Q_0) &\models (\varphi_1 \vee \varphi_2) && \longleftrightarrow (\mathcal{T}, Q_0) \models \varphi_1 \text{ or } (\mathcal{T}, Q_0) \models \varphi_2 \\
(\mathcal{T}, Q_0) &\models \langle\!\langle A \rangle\!\rangle\, (\varphi_1 \,\mathbf{U}_I\, \varphi_2) && \longleftrightarrow \exists F_A\, \forall \zeta \in \mathrm{sim}_{\mathcal{T}}\, (Q_0, F_A)\, \exists i \in \mathbb{N}\, \exists j \in \mathbb{N} \\
& && \quad \exists t_1 \in I\; (\mathcal{T}, \{\zeta(t_1, i)\}) \models \varphi_2 \\
& && \quad \wedge \forall t_2 \in [\inf(I), t_1)\; (\mathcal{T}, \{\zeta(t_2, j)\}) \models \varphi_1
\end{aligned}
$$

from the BA thesis of Sami Kharma, Nov. 2021

Model Checking ASTL on Timed Games

- Region-equivalence construction can be lifted to timed CGS
  - model checking for TATL on timed games is exponential [BLMO 07]
  - our result: lifting results to ASTL
  - still unclear: the case of ASTL*
- ASTL on rectangular hybrid games is still open
  - but we believe that it could be done

	algo. compl. w.r.t. ϕ and $\mathcal{T}$	theoretical complexity
ATL*	$2^{2^{O(\vert\phi\vert)}} \cdot 2^{O(\vert\mathcal{T}\vert)}$	2EXPTIME-complete
TATL	$2^{O(\vert\phi\vert) \cdot O(\vert\mathcal{T}\vert)}$	EXPTIME-complete
TALTL	$2^{2^{O(\vert\phi\vert)}} \cdot 2^{O(\vert\mathcal{T}\vert)}$	2EXPTIME-complete

from Brihaye, Laroussinie, Markey, Oreiby: Timed Concurrent Game Structures

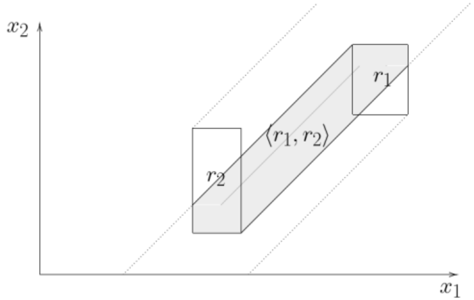
Algorithm

Algorithm 1 ASTL symbolic model-checking

Input: timed game $\mathcal{T}$, ASTL formula φ
Output: boolean *true* or *false*

for φ' **in** $\mathrm{Sub}(\varphi)$ **do**
    case $\varphi' = \top$
        $[\varphi'] \leftarrow Q_I$
    case $\varphi' = x \sim c$
        $[\varphi'] \leftarrow \mathrm{Reg}_{\mathcal{T}}(x \sim c)$
    case $\varphi' = \neg\theta$
        $[\varphi'] \leftarrow Q_I \setminus [\theta]$
    case $\varphi' = (\theta_1 \vee \theta_2)$
        $[\varphi'] \leftarrow [\theta_1] \cup [\theta_2]$
    case $\varphi' = \langle\langle A \rangle\rangle\, (\theta_1\, \mathbf{U}_I\, \theta_2)$
        $[\varphi'] \leftarrow \mathrm{Pre}^*_{\mathcal{T},I}(A, [\theta_2], [\theta_1])$
end for
return $q_0 \in [\varphi]$

- Pre-image calculation: for a timed game, set of players, interval, returnes the set of states from which a goal state can be reached with one decision, traversing only given intermediate states
- can be calculated by algebraic considerations

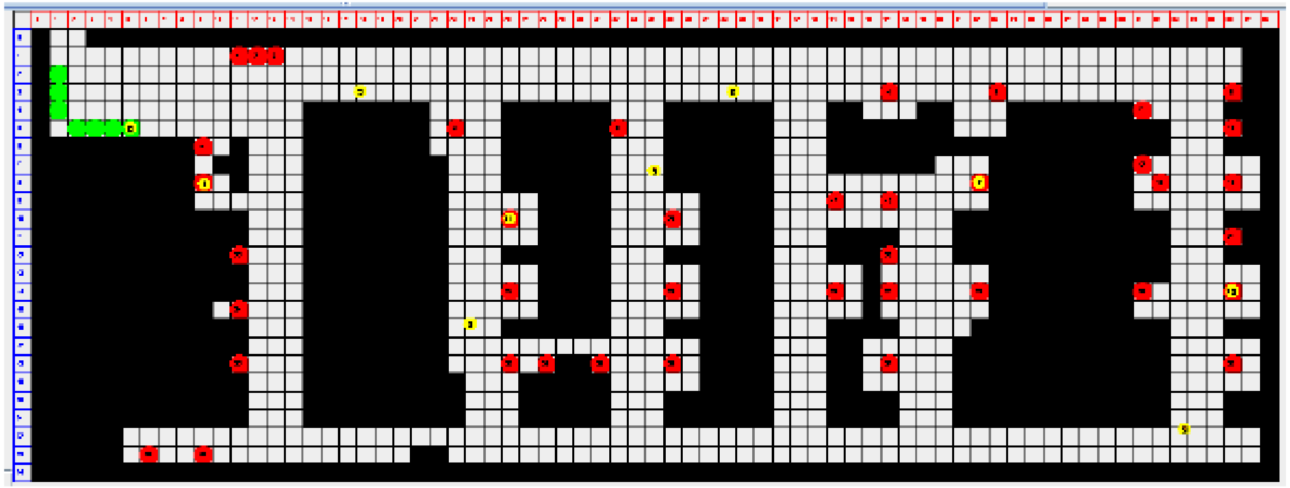


Experimental Results

- Yet unavailable

Back to the Transport Robot Example

- MCMAS model
- Robots have a joint strategy to accomplish all transport jobs in time: checked
- „Fuzzy" properties
  - Average and maximal waiting time should be as low as possible
  - Robots should keep their battery charged between 40% and 60%, if possible
  - Robots should provides approximately equal wear and tear within the fleet
  - Target properties are in Pareto equilibrium
  - "Reasoning about Quality and Fuzziness of Strategic Behaviours" [Bouyer et al. 2019]



Conclusion

- Presented a new combination of strategic and continuous logic
  - model checking algorithm
- Well-suited to specify and model certain properties
  - real time, interactivity
- Models for industrial control tasks are often much more complex
  - no firm goals, but approximate targets

- Vision: Verify control program with respect to the objectives
- Dream: Generate control programs automatically from the rules

Thank you for your attention!