# On the Essence and Initiality of Conflicts

Guilherme Grochau Azzi[1], *Andrea Corradini*[2]
and Leila Ribeiro[1]

[1]Instituto de Informática
Universidade Federal do Rio Grande do Sul, Porto Alegre, Brazil

[2]Dipartimento di Informatica
Università di Pisa, Pisa, Italy

IFIP WG1.3 Foundations of System Specification
Prague, April 6-7, 2019

*based on a paper presented at the 11th International Conference on Graph Transformation, June 2018*

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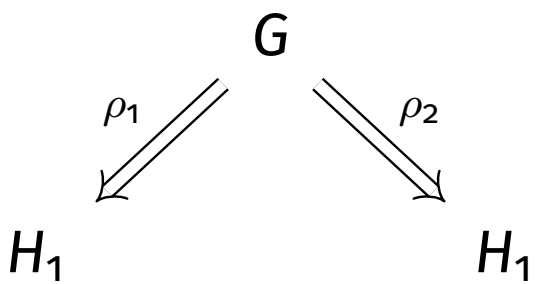
- Intrinsic non-determinism of the modeled system could be mixed with the one arising from the rule-based specification of functional transformations.
- Analysis of conditions for independence and for potential conflicts among transformations becomes fundamental for the analysis of such systems.

# Parallel independence vs. conflicts

Since (linear) graph transformation is resource-conscious, “confluence” is “strict confluence”.

## Parallel Independence
### of Transformations



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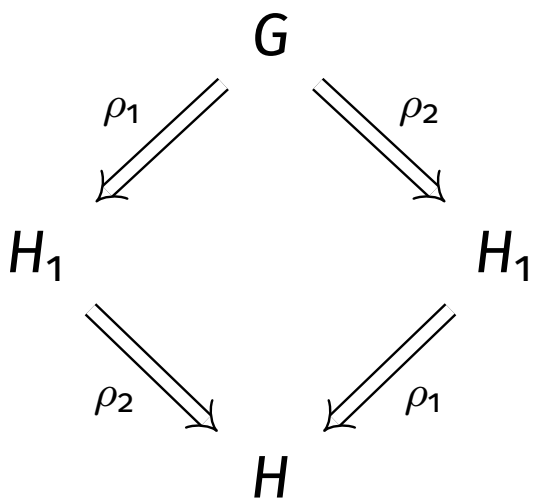

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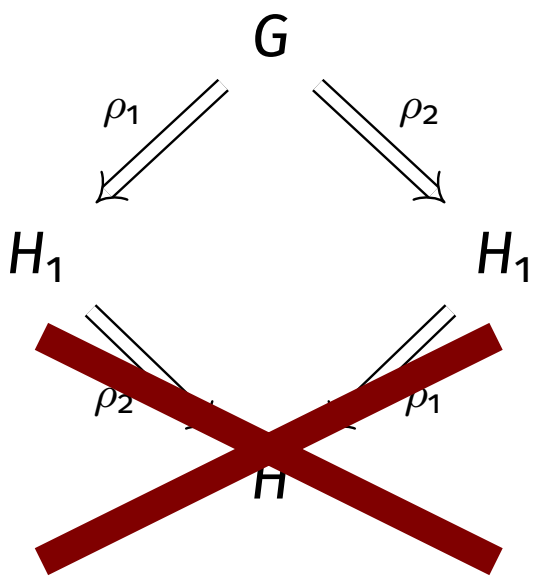


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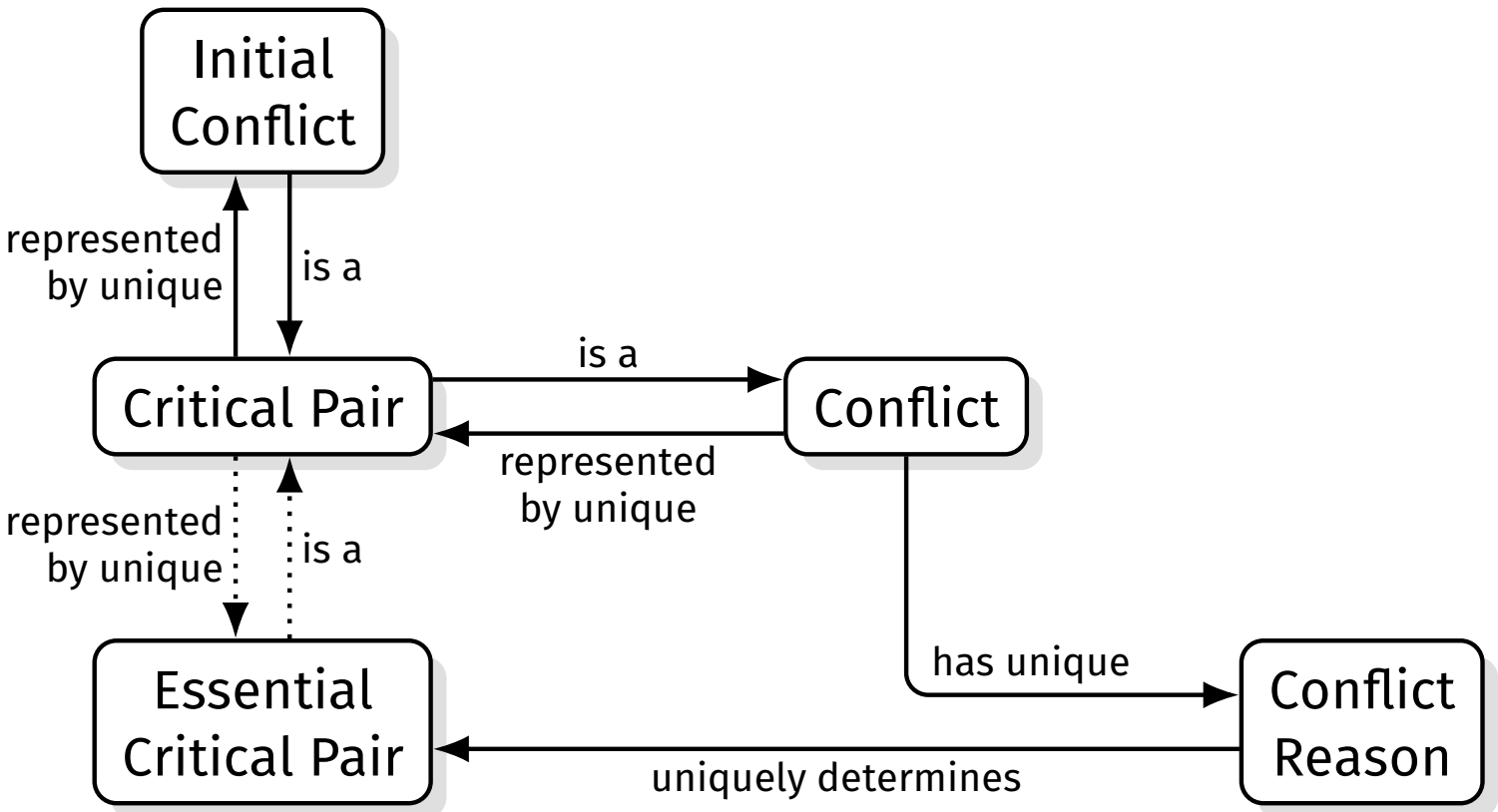
- “Root causes” are the resources for which the transformation compete

# Overview of previous results

Available for: —— Adhesive Categories - - - - $\mathbb{S}\mathrm{et}^{\mathbb{S}}$ ····· $\mathbb{G}\mathrm{raph}_T$



# Overview of our results in context

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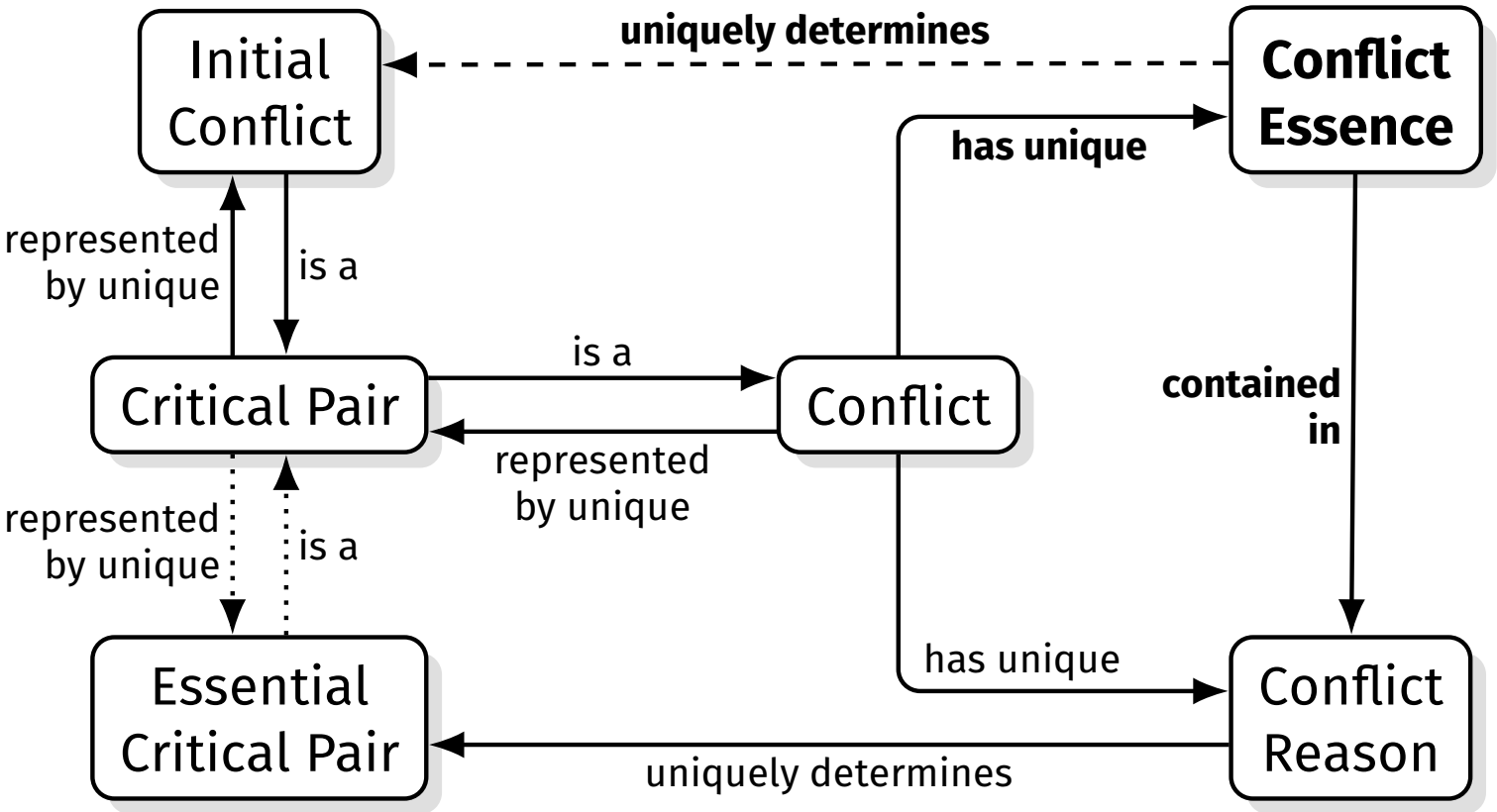


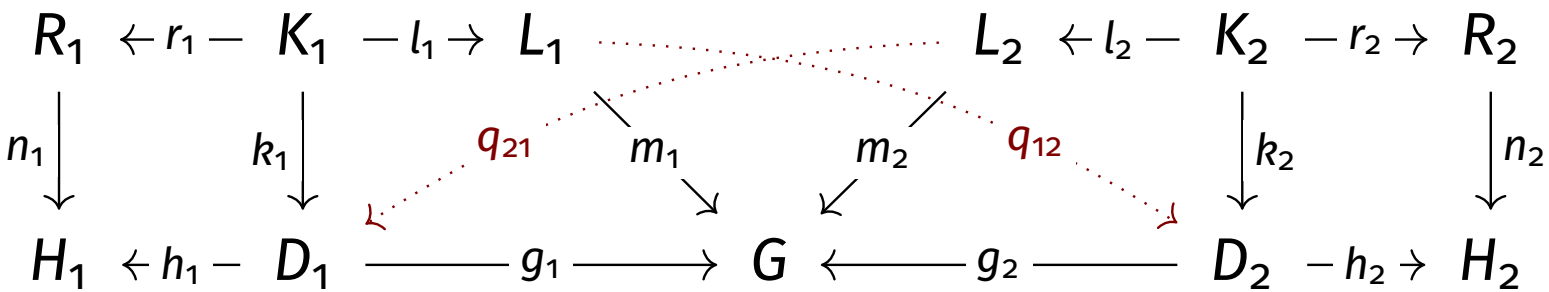
# Background: The DPO Approach

$$\begin{aligned} \text{Rule:} \quad & \rho = L \overset{l}{\rightarrowtail} K \overset{r}{\rightarrowtail} R \\ \text{Match:} \quad & m : L \rightarrowtail G \\ \text{Transformation:} \quad & G \overset{\rho,m}{\Longrightarrow} H \end{aligned}$$

$$\begin{array}{ccccc} L & \xleftarrow{\ l\ } & K & \xrightarrow{\ r\ } & R \\ \downarrow{\scriptstyle m} & \text{PO} & \downarrow{\scriptstyle k} & \text{PO} & \downarrow{\scriptstyle n} \\ G & \xleftarrow[\ g\ ]{} & D & \xrightarrow[\ h\ ]{} & H \end{array}$$

# New Perspective on Parallel Independence

- Previous work based on the **standard condition** for parallel independence



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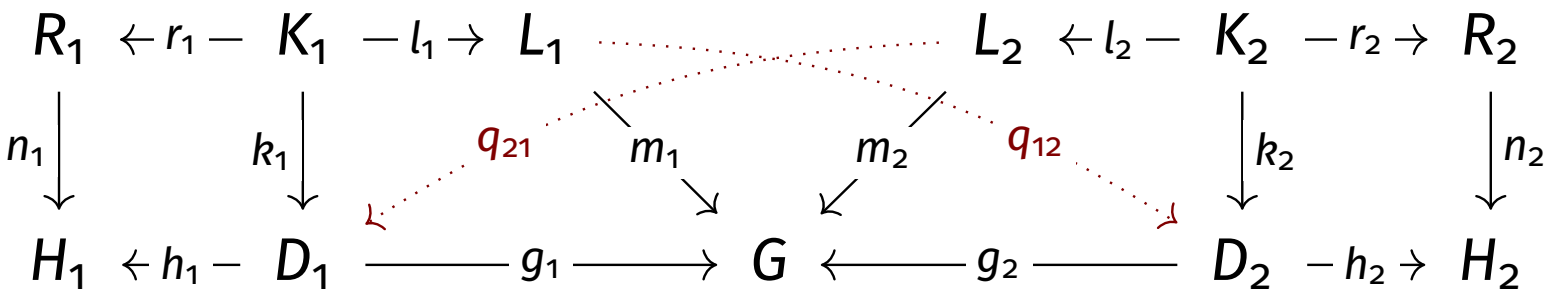


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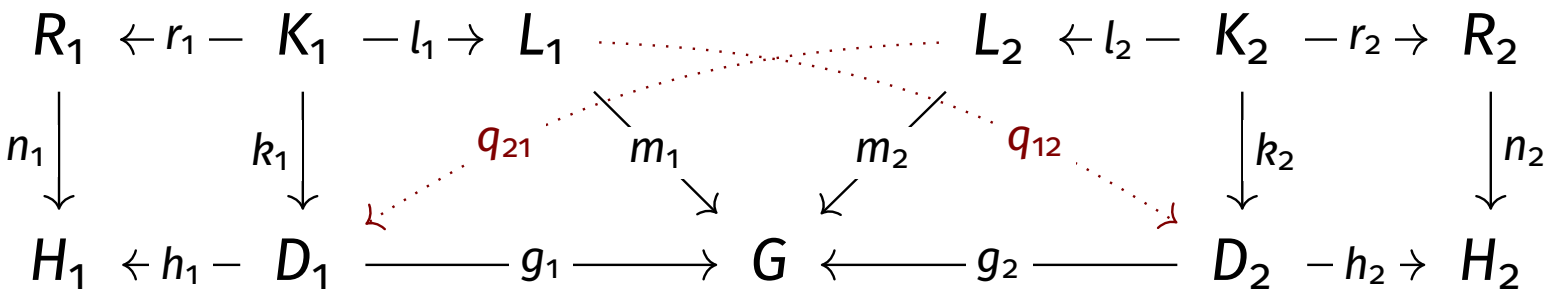


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- **Goal:** review characterization of conflicts under new light

# Background: Adhesive Categories

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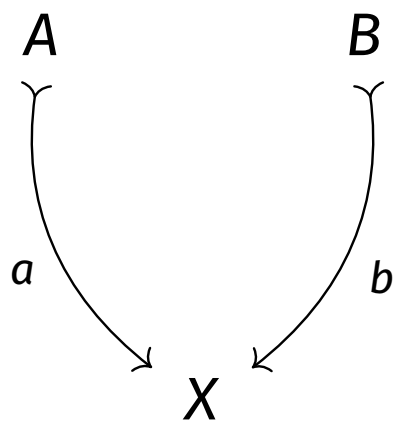

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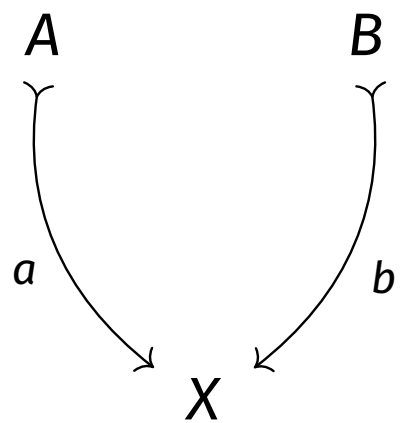

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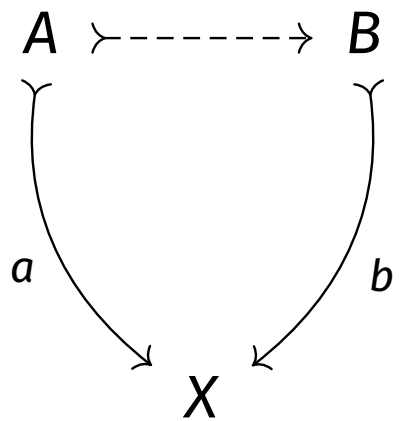

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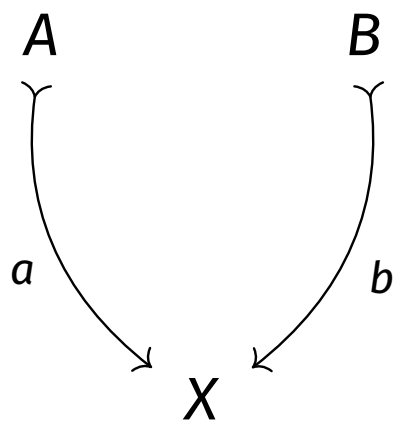

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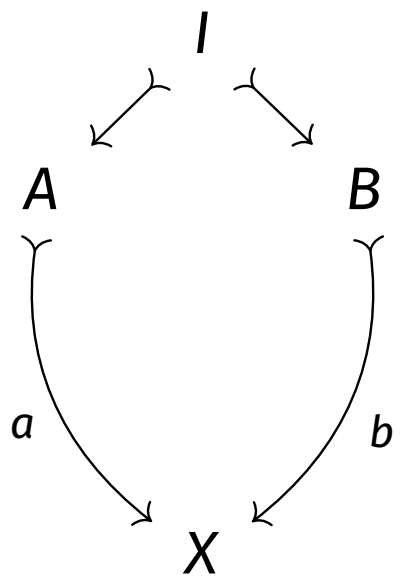

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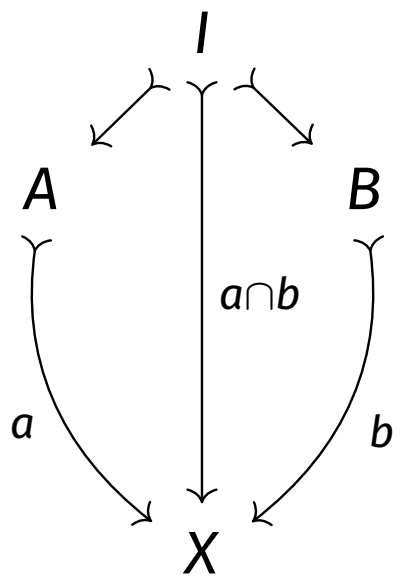

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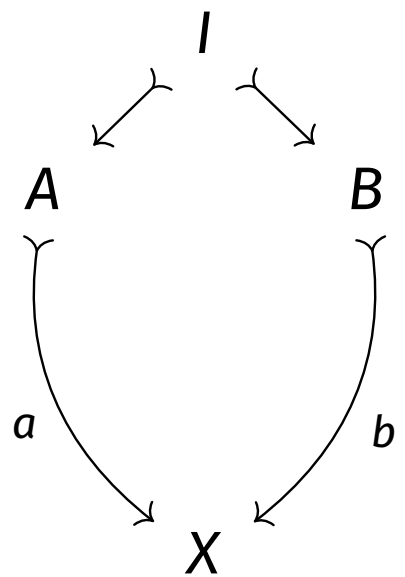

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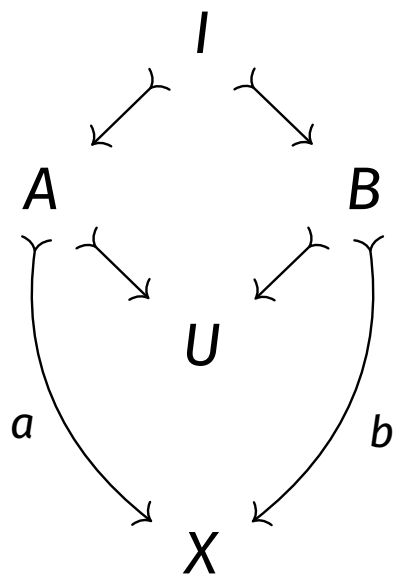

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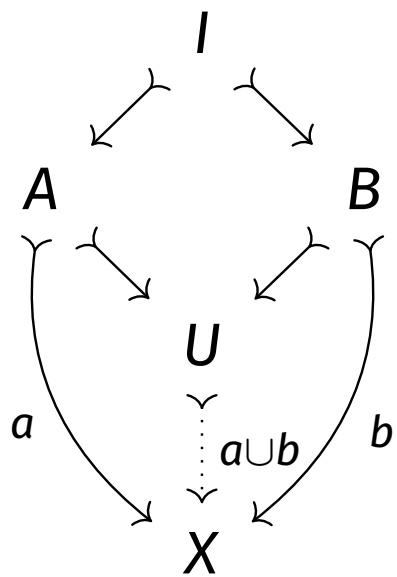

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- Always adhesive

# Outline

1. Characterize conflict between transformations
2. Useful properties of the characterization
3. Compare with conflict reasons of Lambers, Ehrig, and Orejas (2008)
4. Relate to initial conflicts

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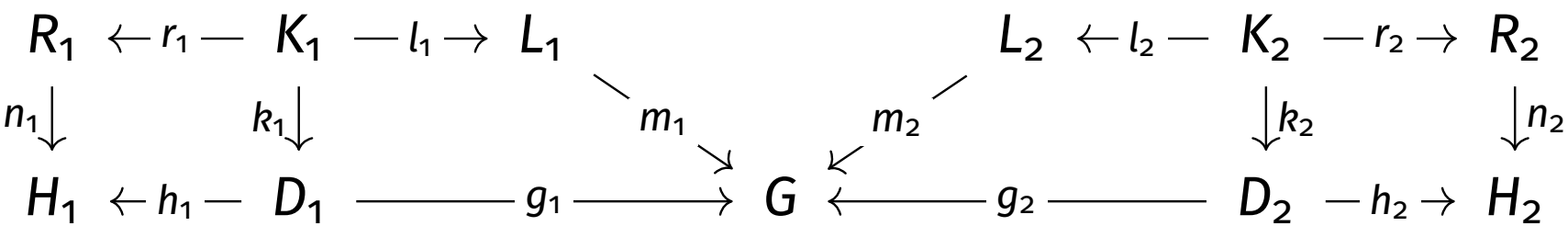
$$H_1 \overset{t_1}{\Longleftarrow} G \overset{t_2}{\Longrightarrow} H_2$$



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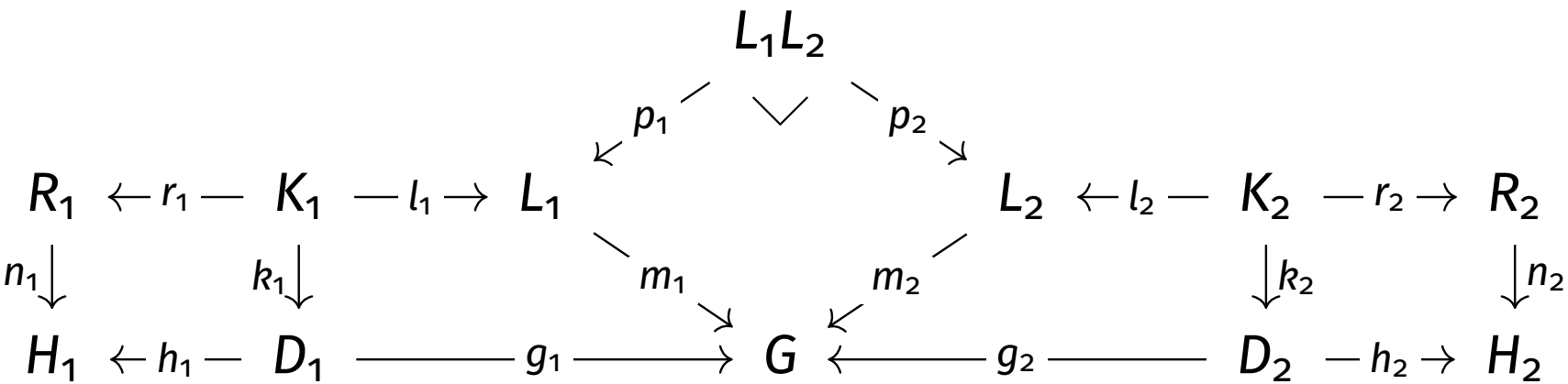

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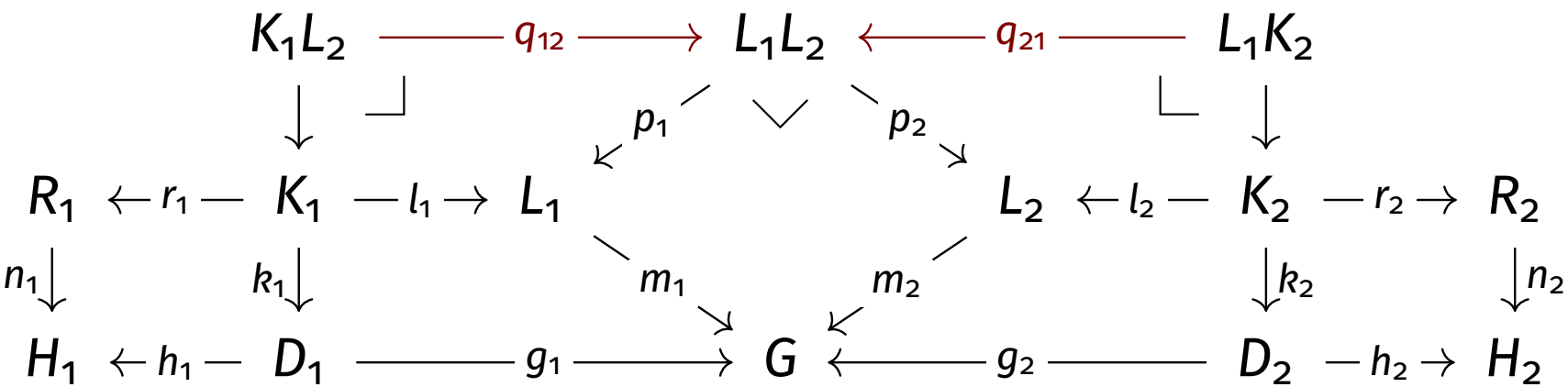

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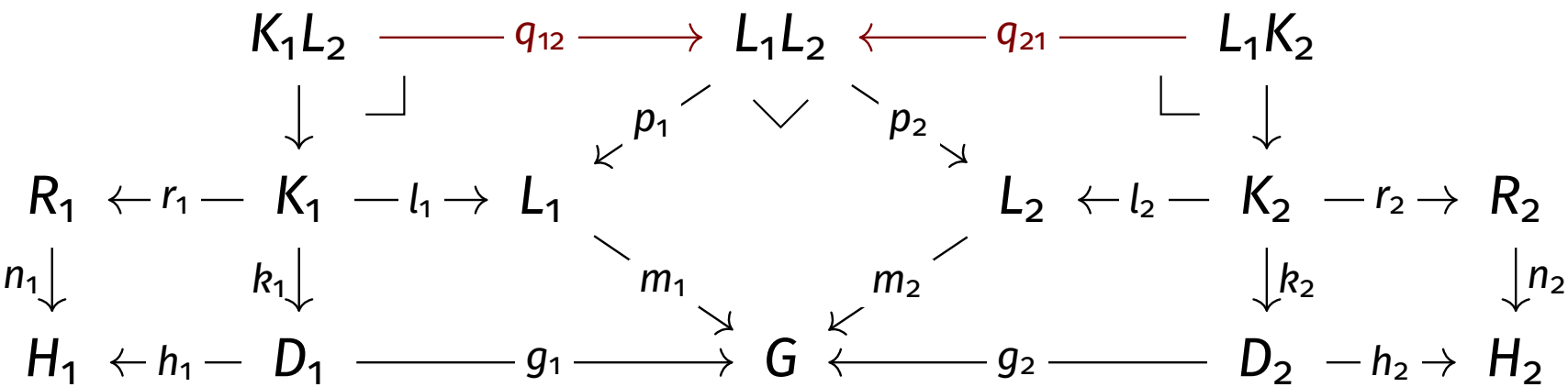


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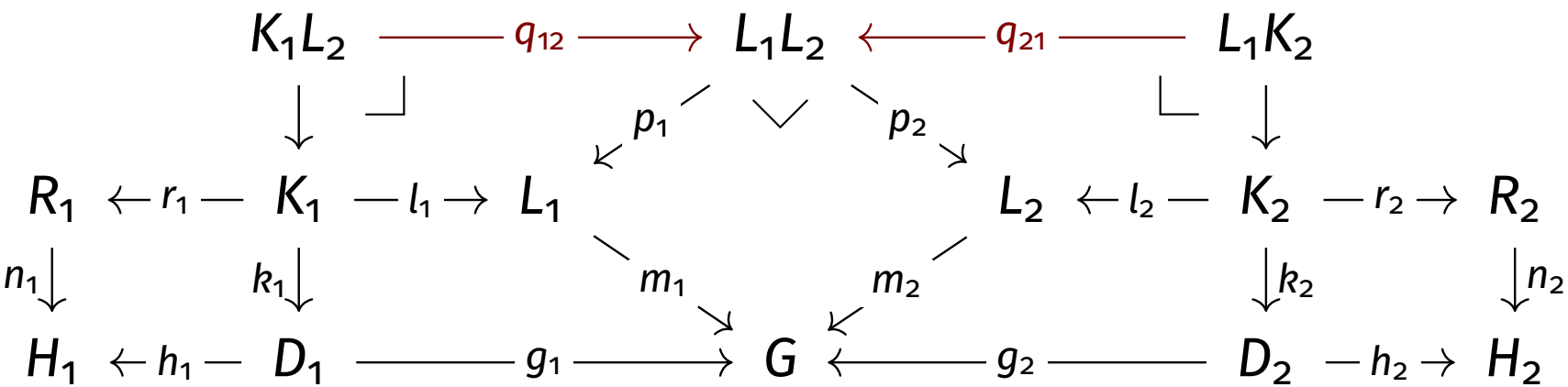


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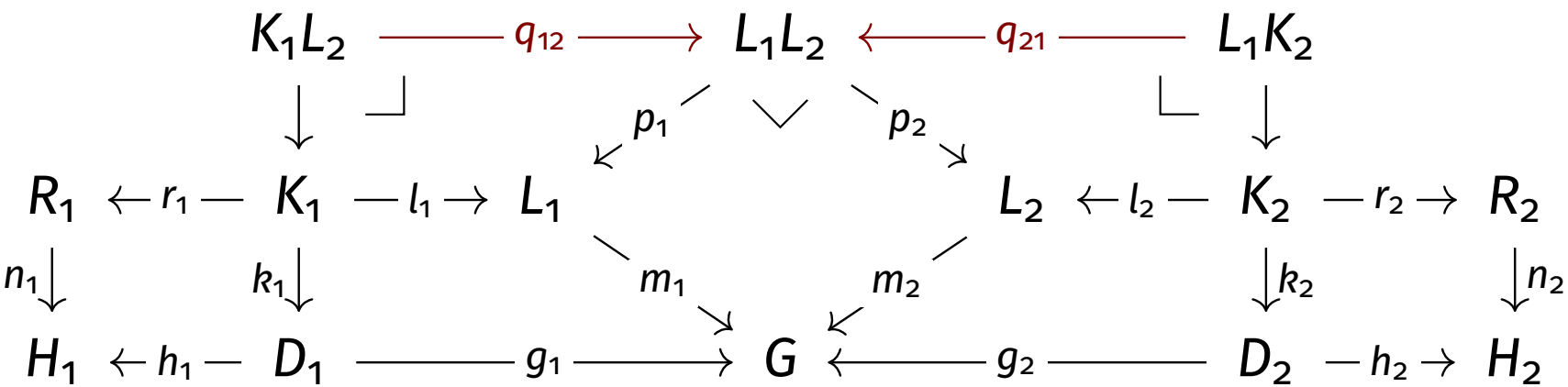


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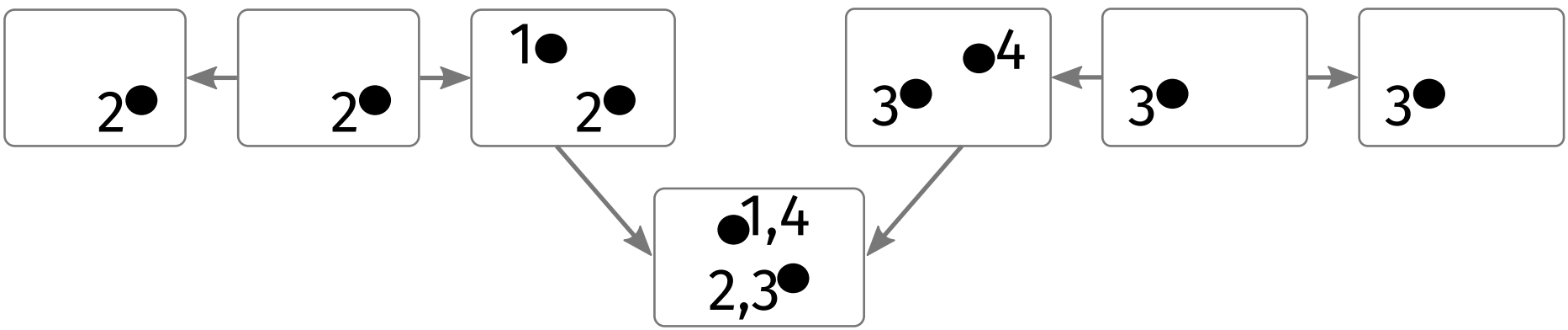
- $K_1L_2 \to L_1L_2$ not iso $\Rightarrow$ $t_1$ disables $t_2$

# Example: Conflict



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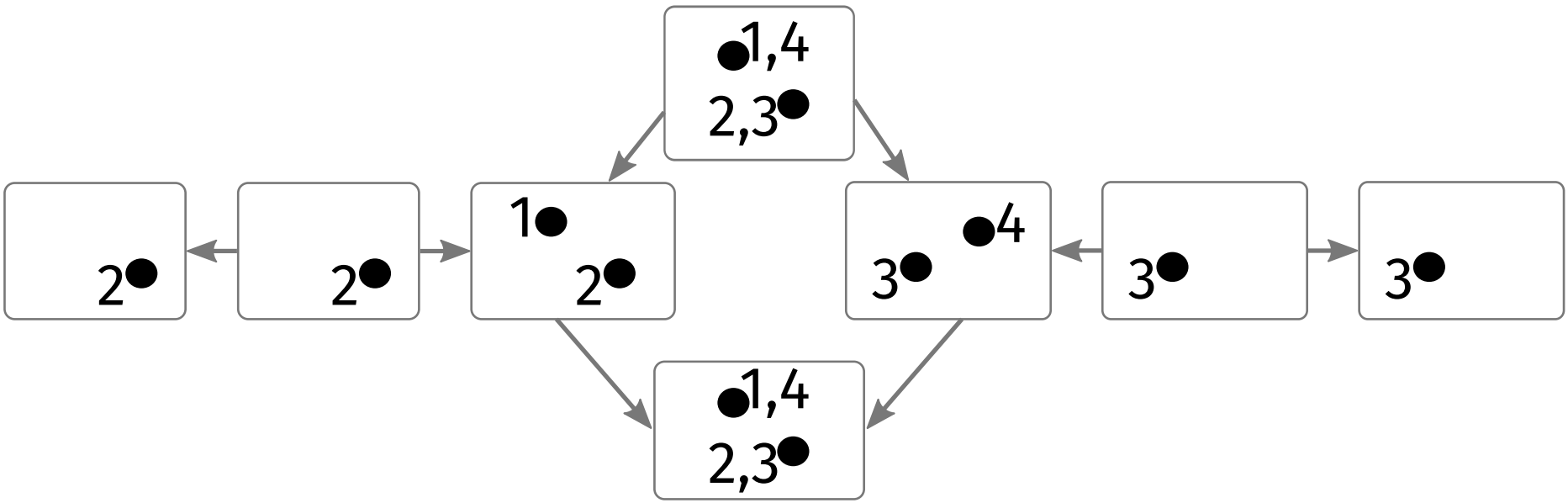

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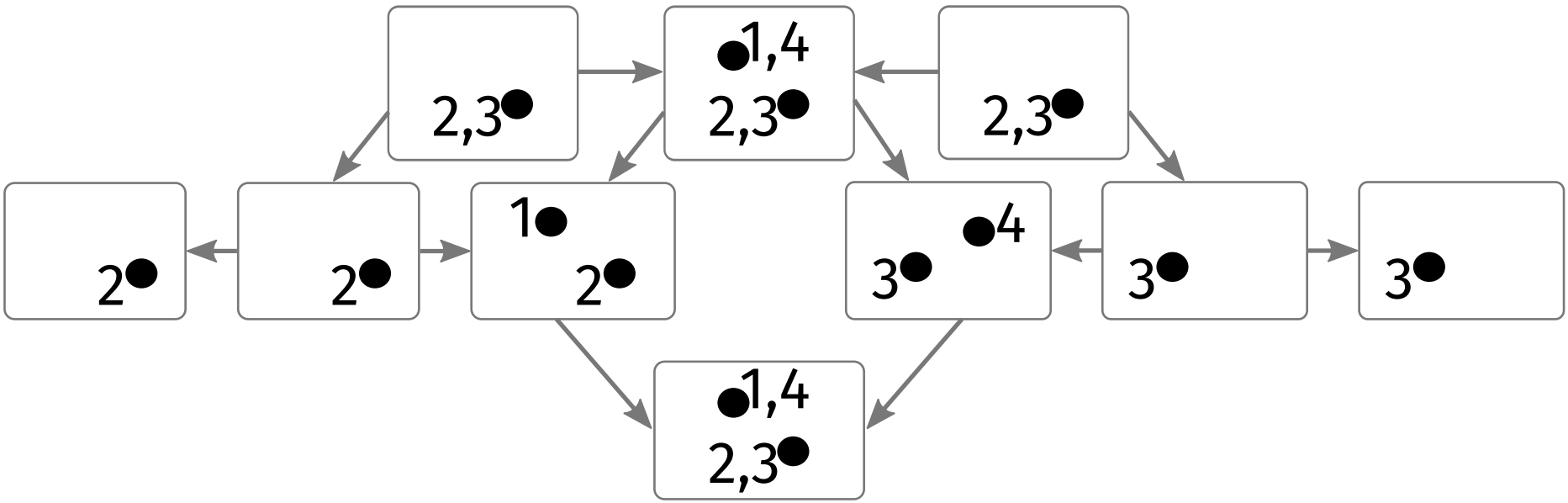

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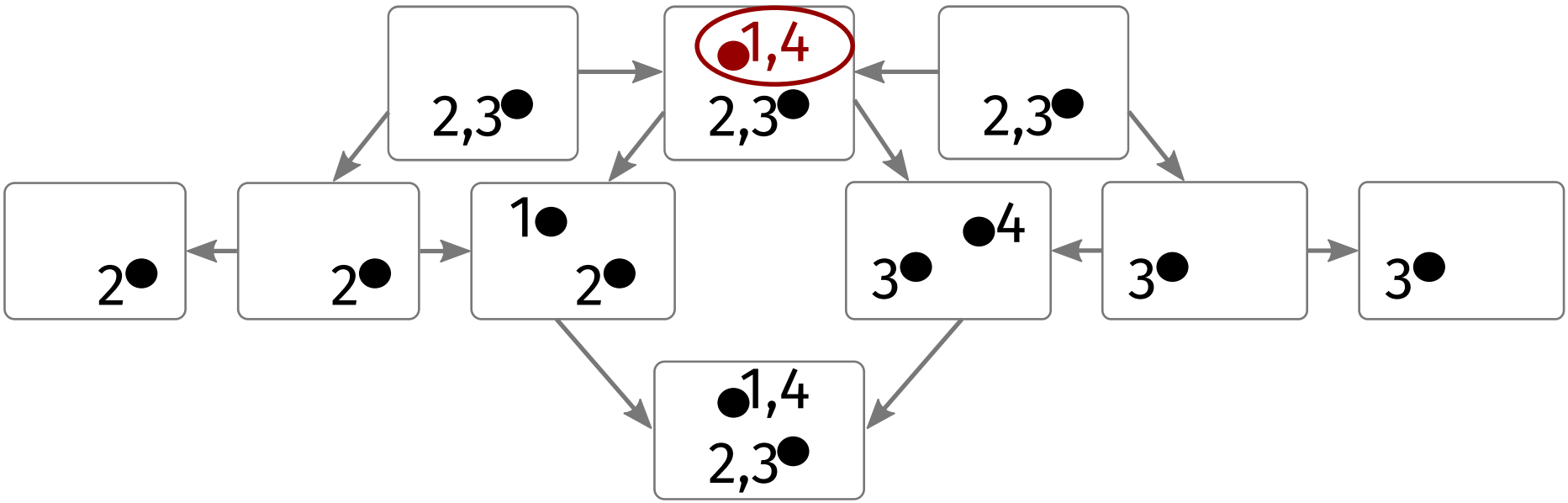

# Determining the Root Cause

- Useful concept: initial pushout over $f : X \to Y$

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- "Categorical diff" for a morphism
- Context $c : C \rightarrowtail Y$ contains "modified stuff"
- Boundary $b : B \rightarrowtail C$ contains "points of contact"

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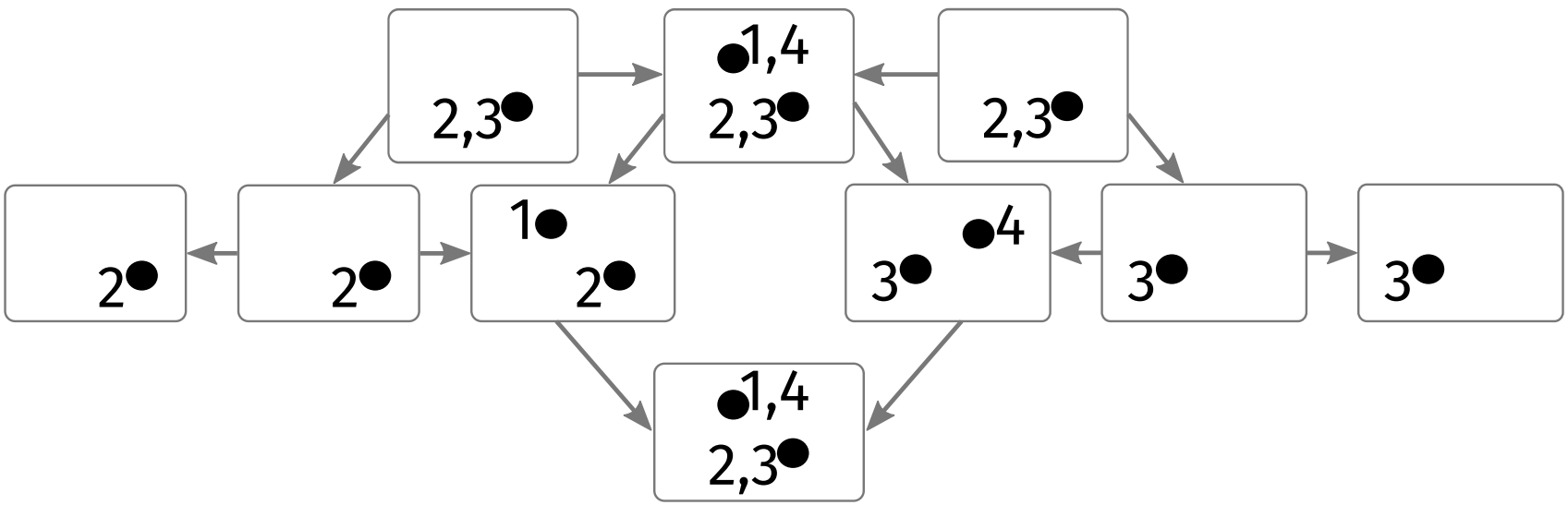

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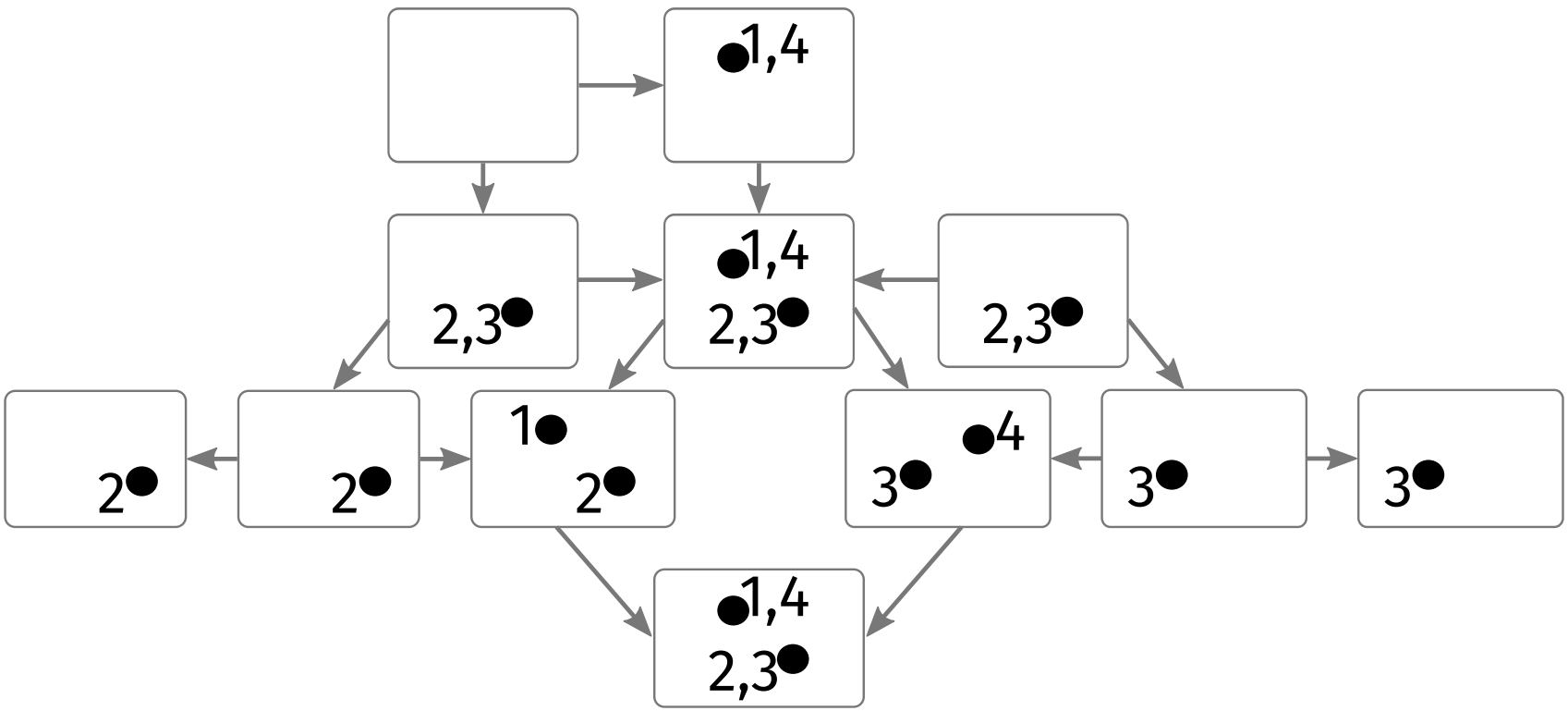

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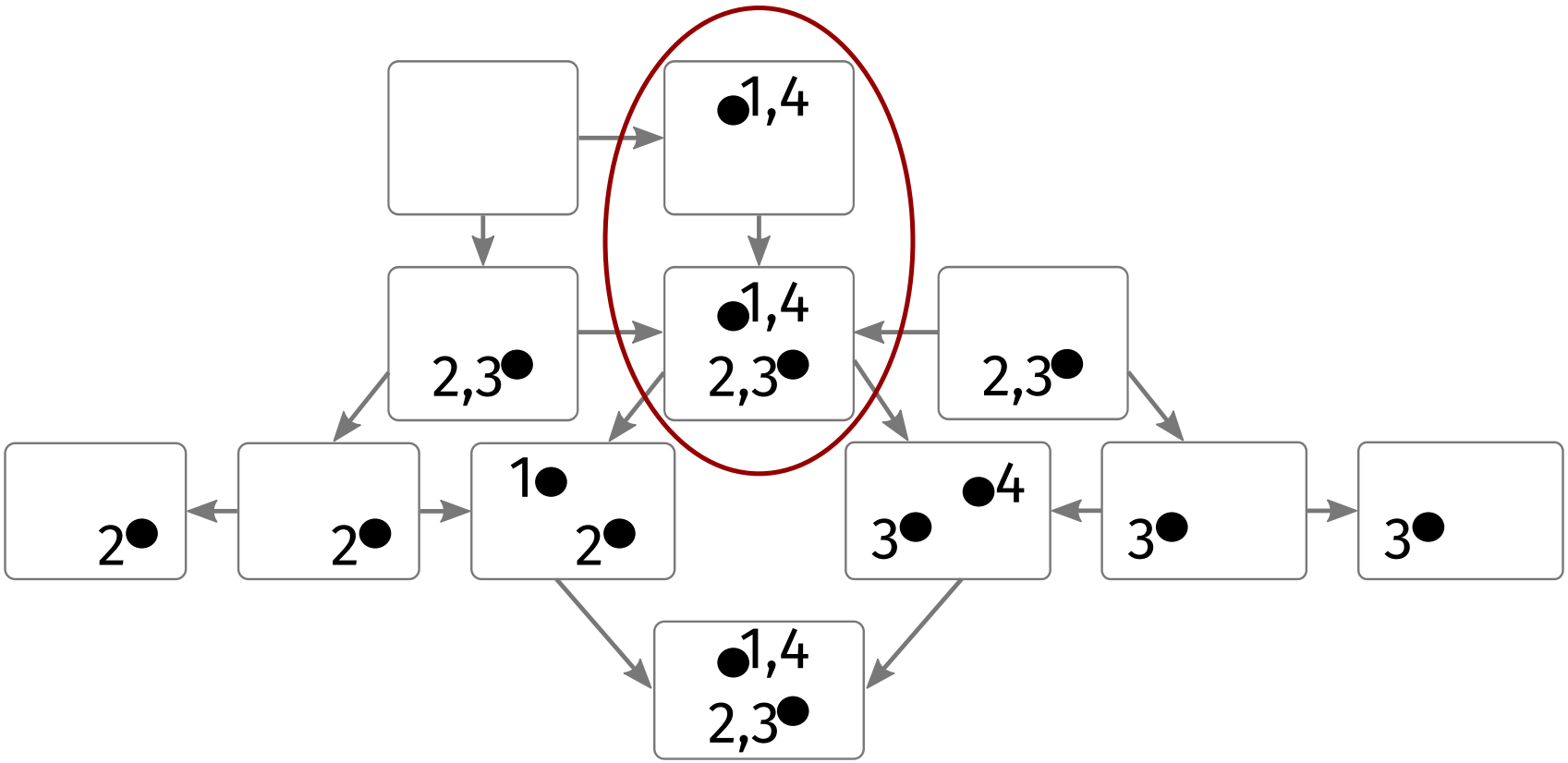

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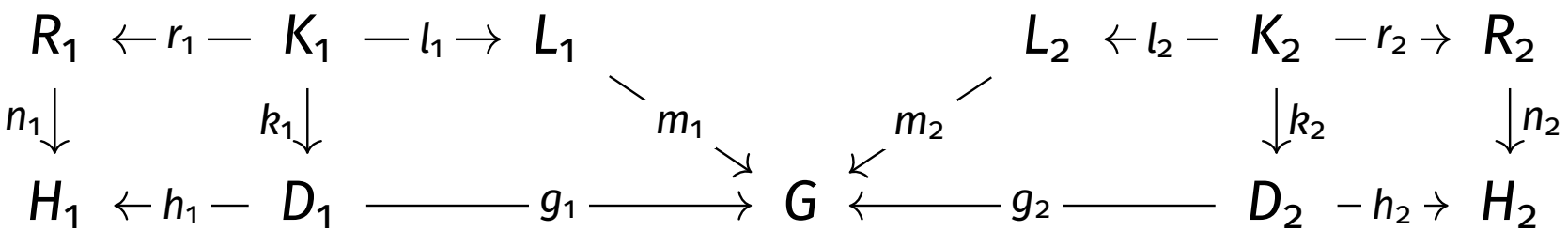

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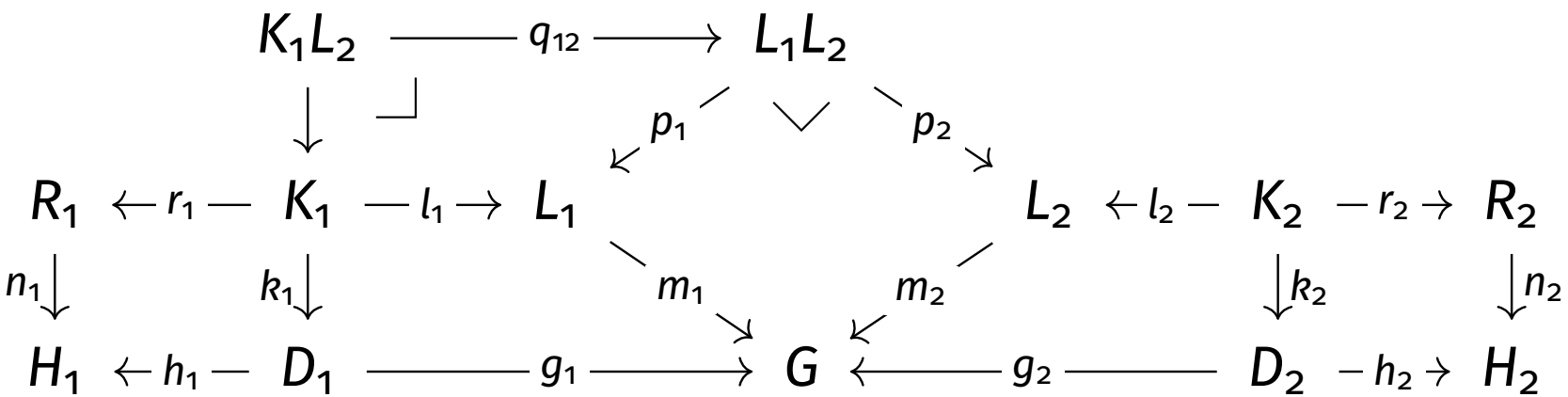

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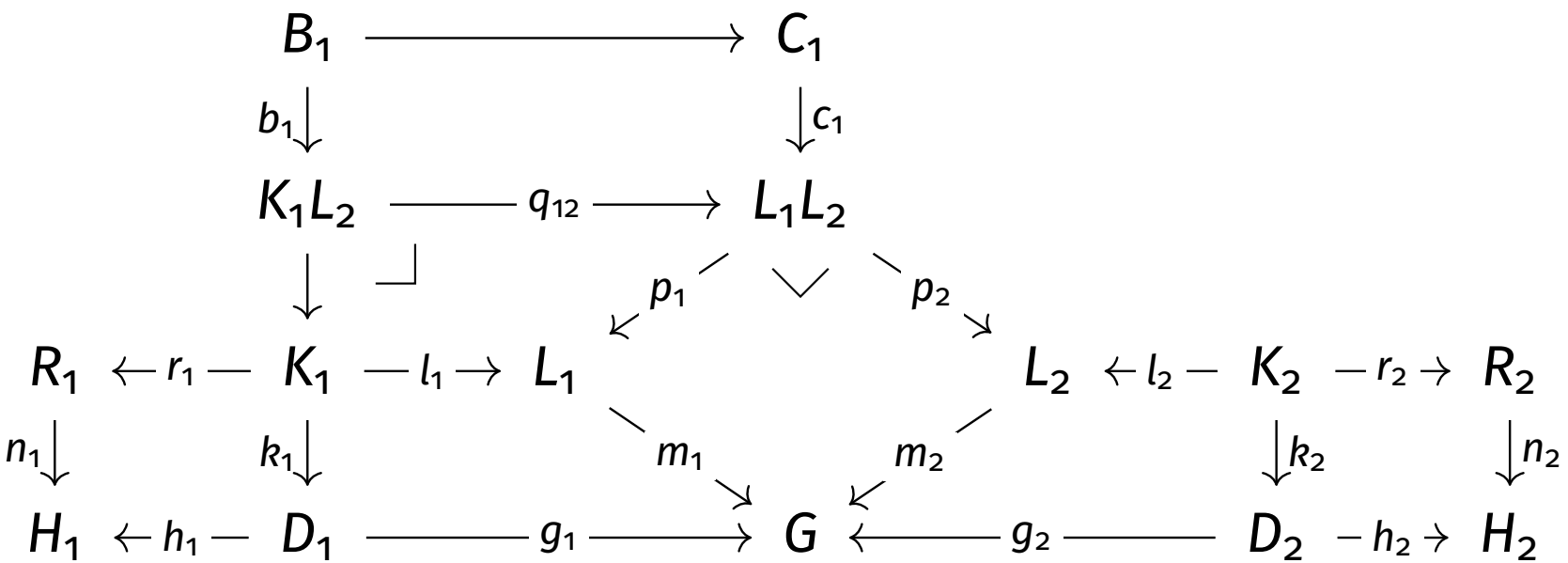

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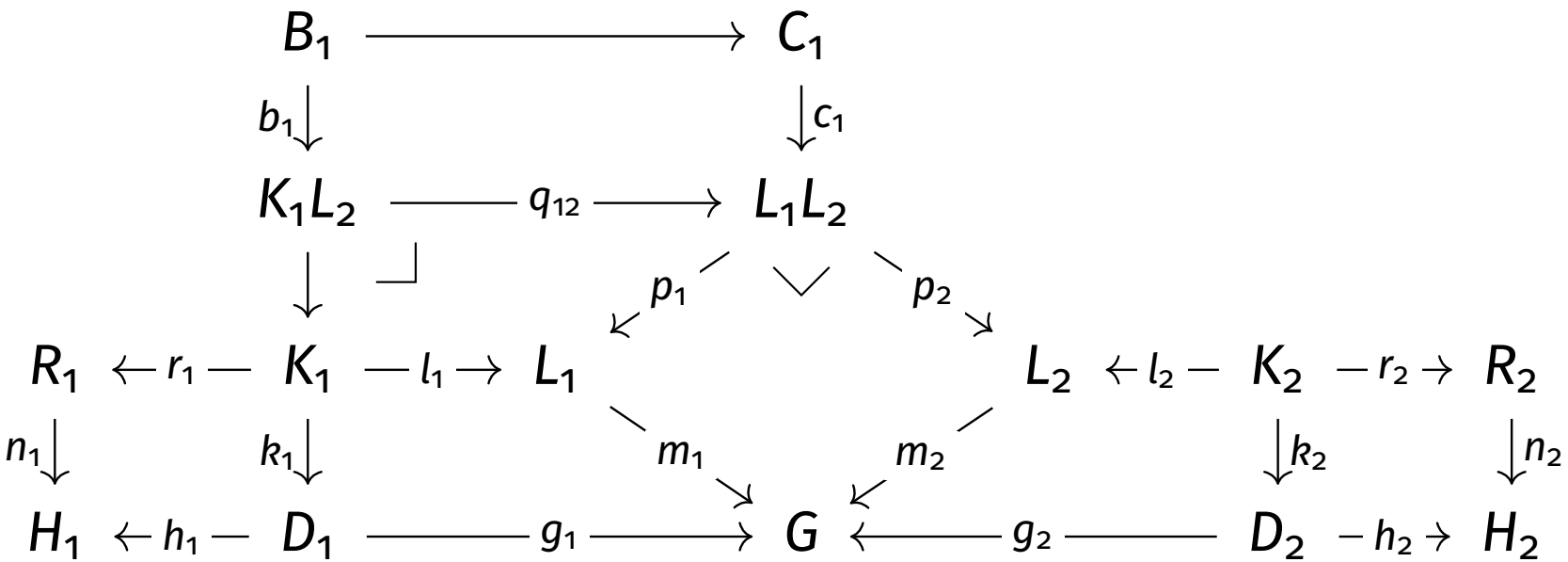

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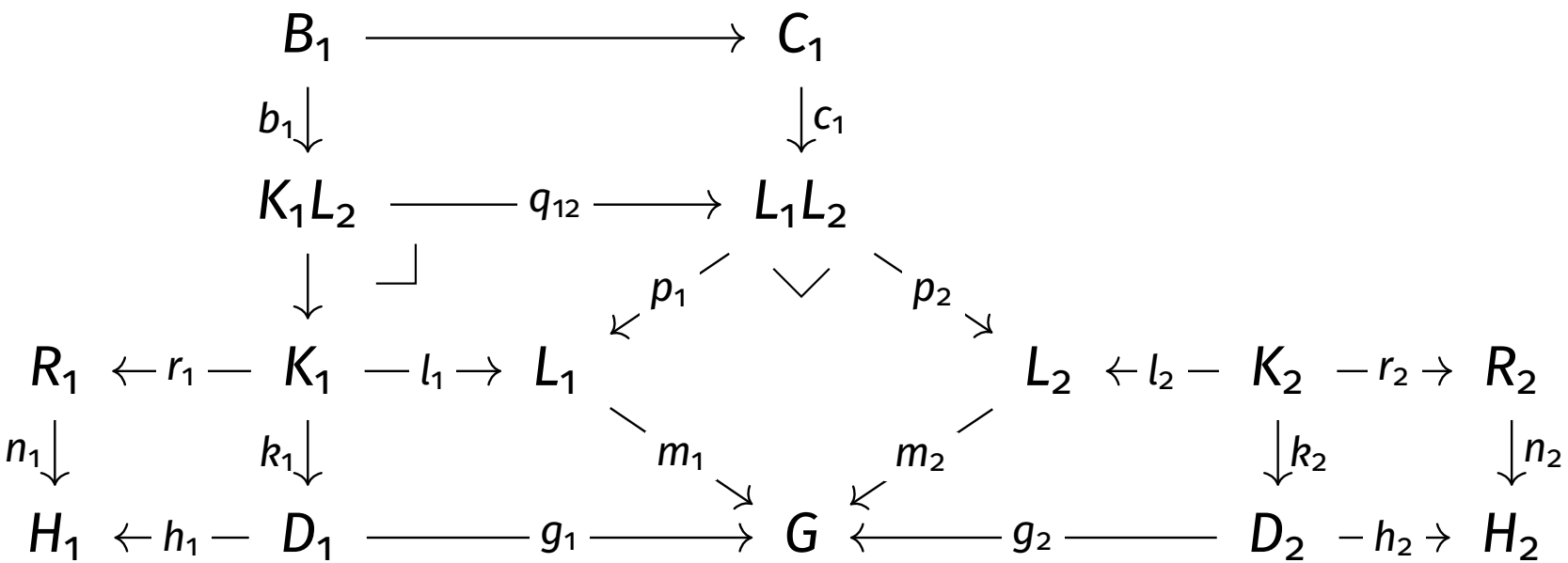

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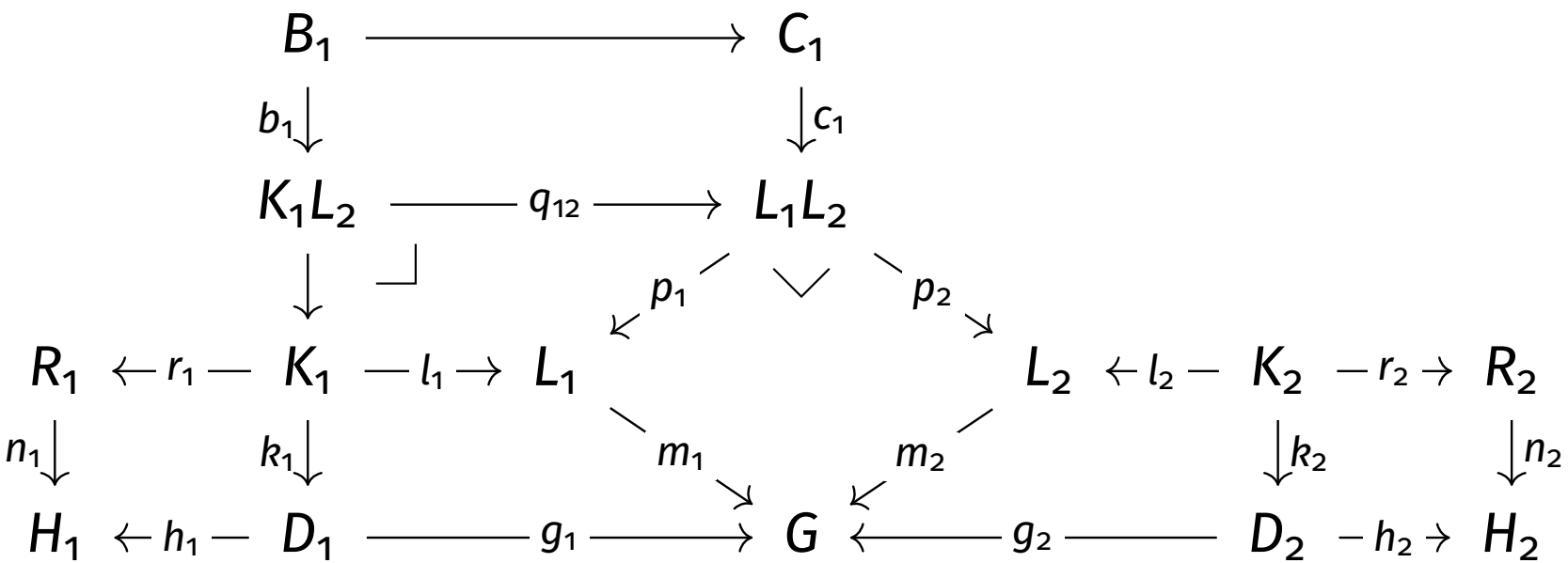
- Conflict essence for $(t_1, t_2)$ is $c = c_1 \cup c_2$



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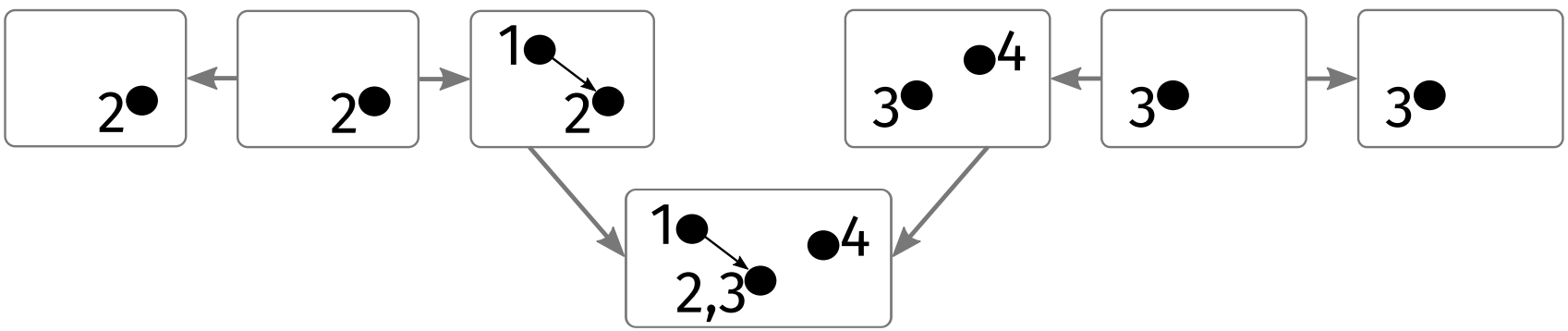

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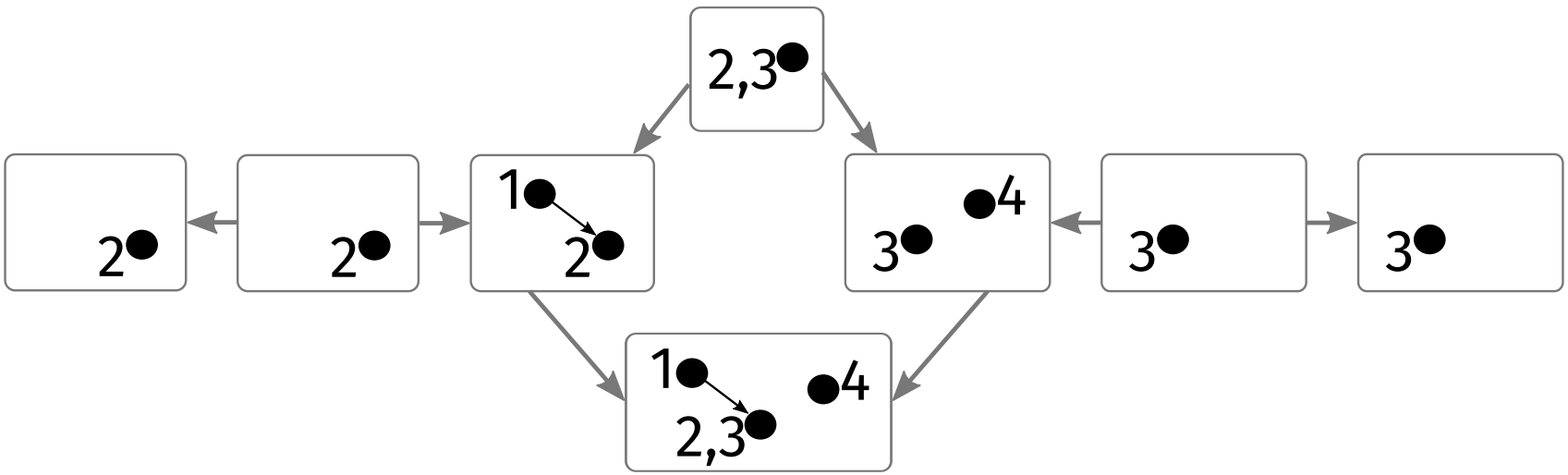

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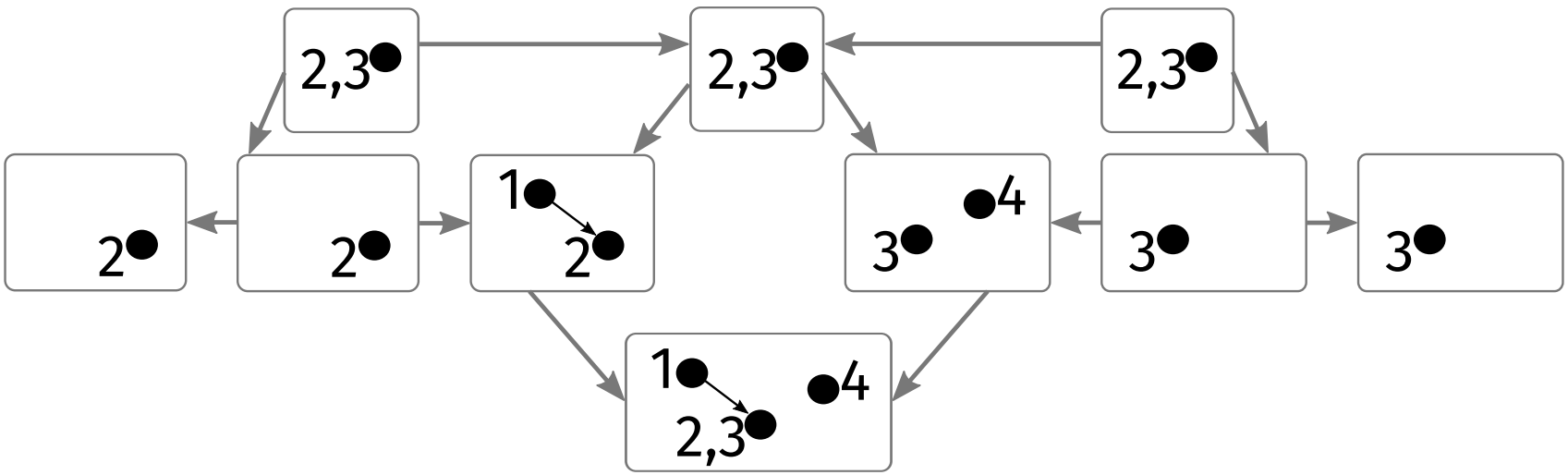

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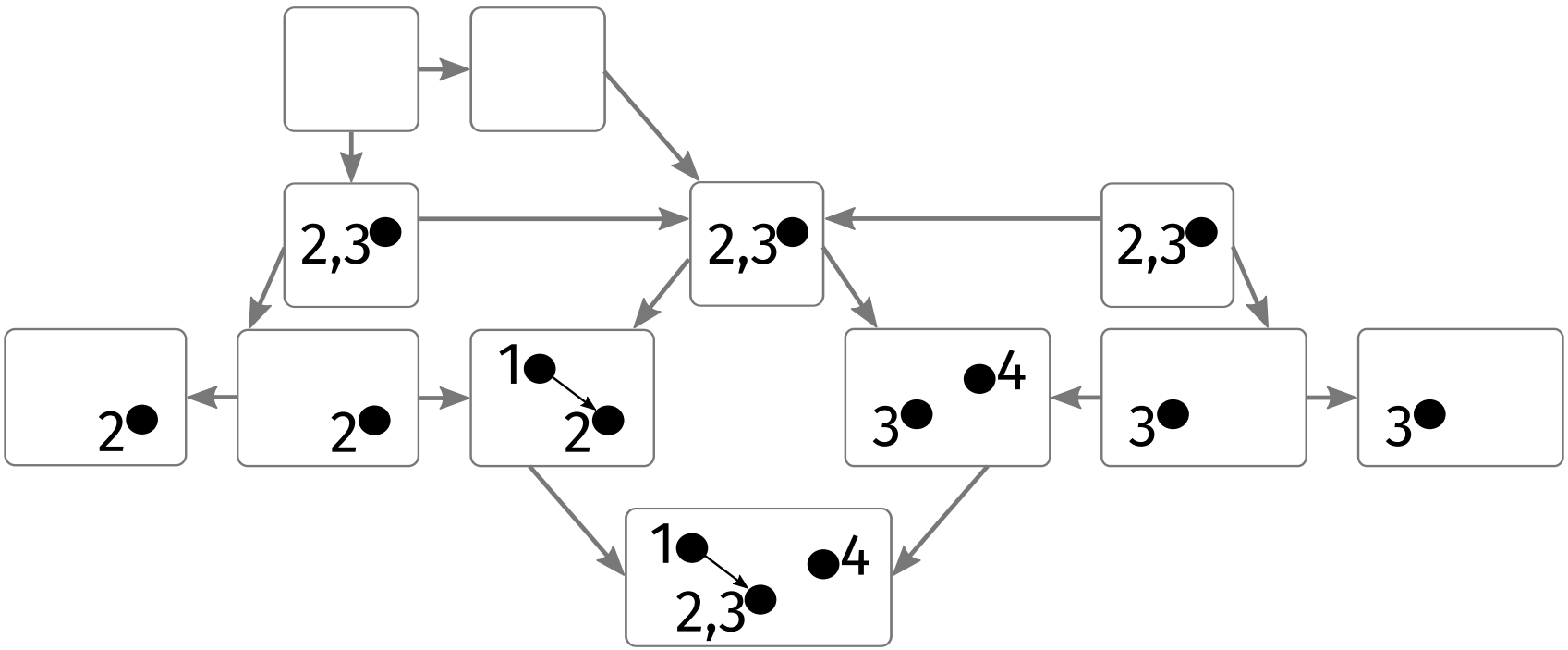

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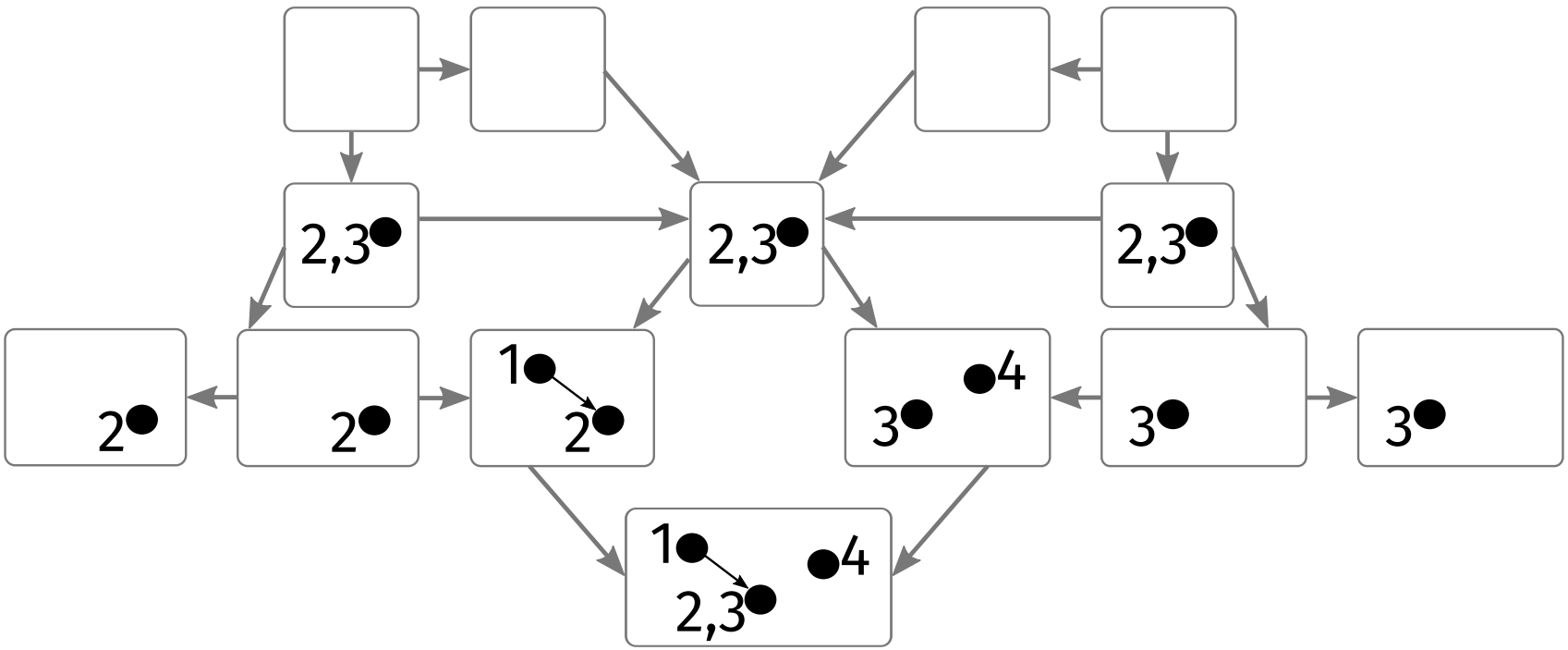

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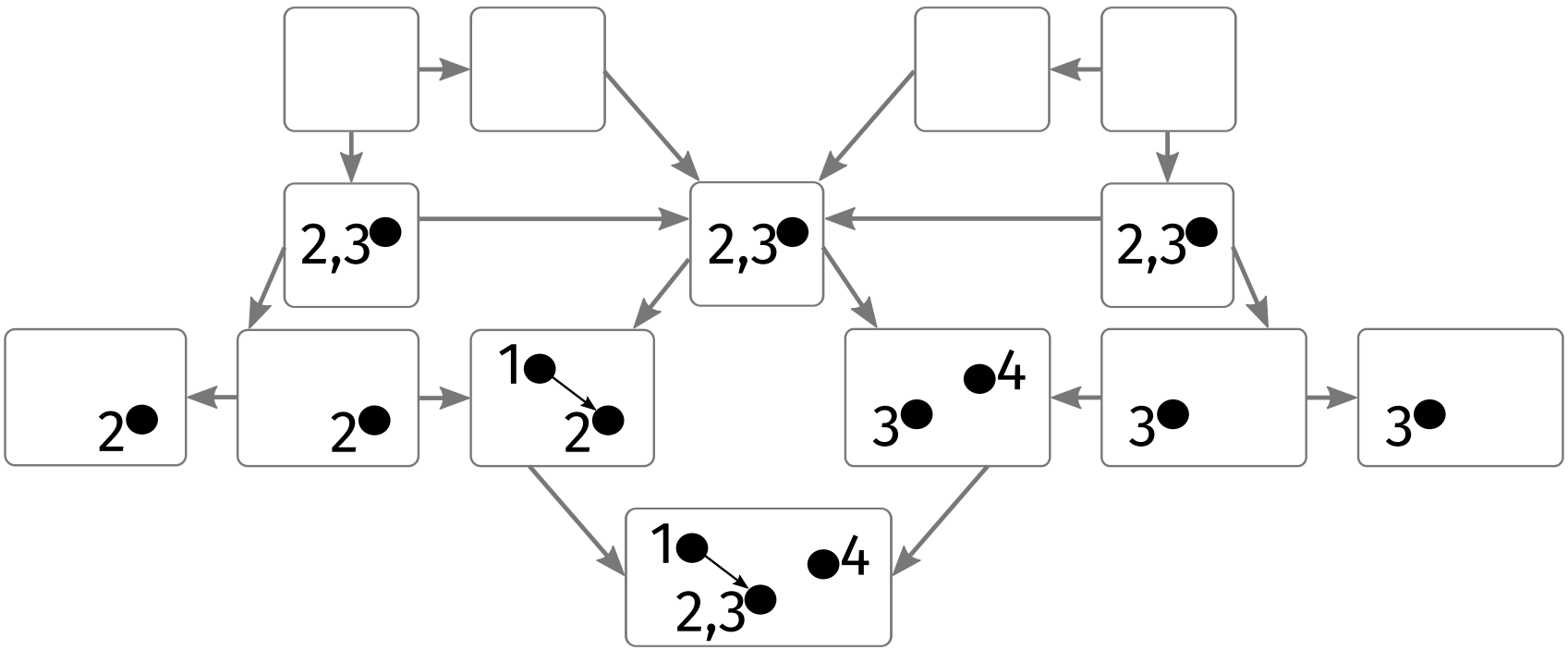


No conflict $\Longrightarrow$ no element caused a conflict

# Empty Essences

Recall: bottom subobject generalizes “emptiness”

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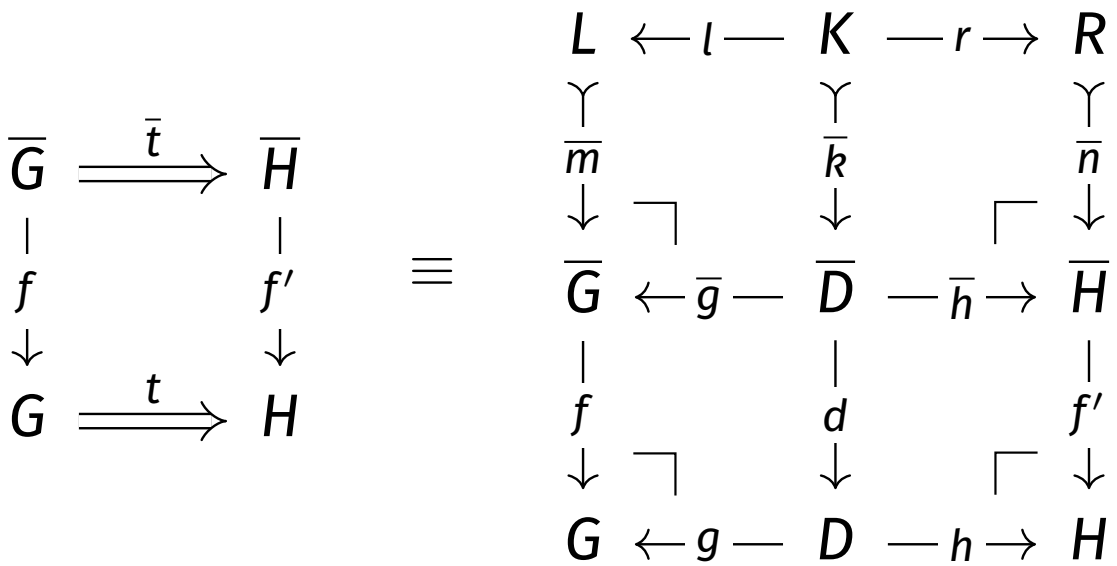
Consider $(t_1, t_2) : H_1 \overset{\rho_1, m_1}{\Longleftarrow} G \overset{\rho_2, m_2}{\Longrightarrow} H_2$

**Theorem**

*The conflict essence for* $(t_1, t_2)$ *is* $\bot \in \mathbf{Sub}(L_1 L_2)$
*if and only if*
$t_1$ *and* $t_2$ *are parallel independent.*

# Extension

- Same transformation in “larger context”



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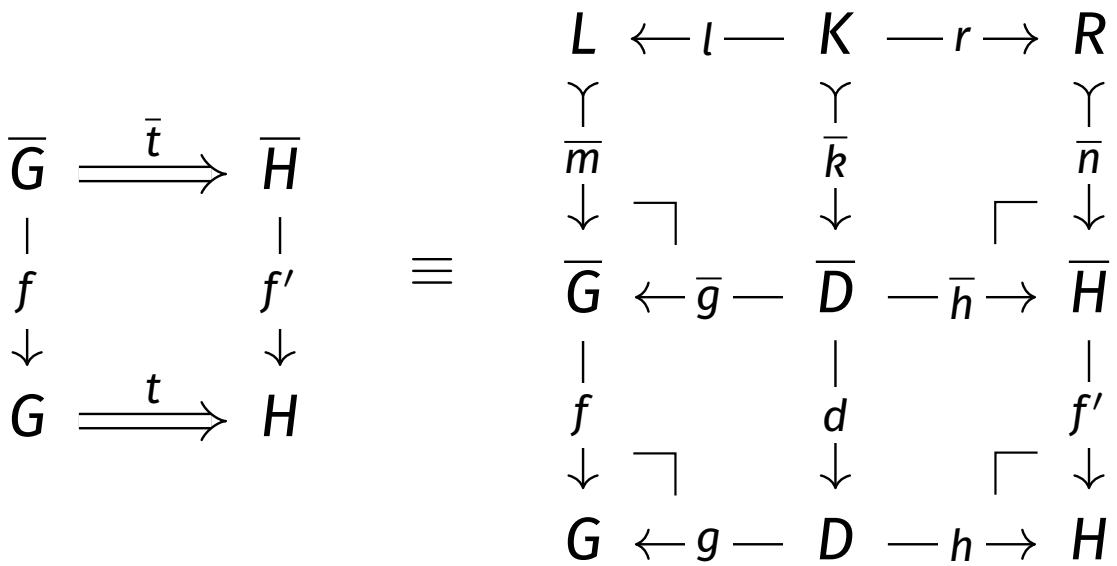


- Lower pushouts ensure $t$ behaves like $\overline{t}$

# Essence Inheritance

**Theorem**

*If extension diagrams below exist,* $(t_1, t_2)$ *and* $(\overline{t_1}, \overline{t_2})$ *have the same disabling and conflict essences.*

$$\begin{array}{ccccc} \overline{H_1} & \xleftarrow{\overline{t_1}} & \overline{G} & \xrightarrow{\overline{t_2}} & \overline{H_2} \\ \downarrow & & \downarrow f & & \downarrow \\ H_1 & \xleftarrow{t_1} & G & \xrightarrow{t_2} & H_2 \end{array}$$

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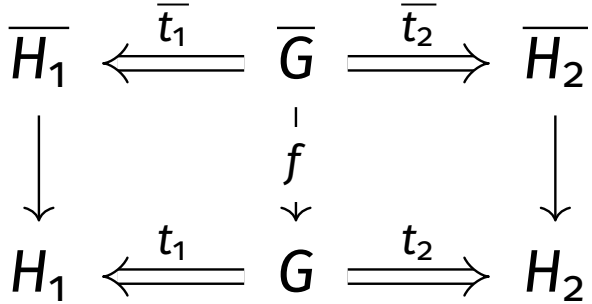


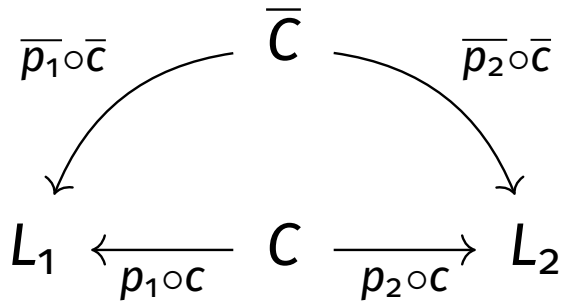

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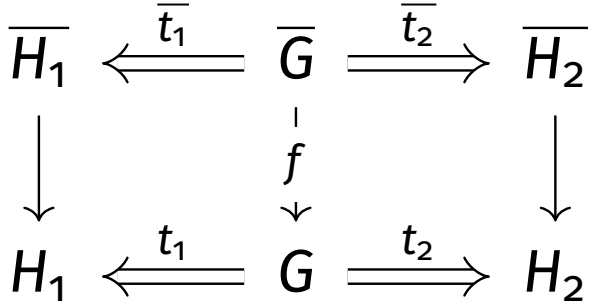


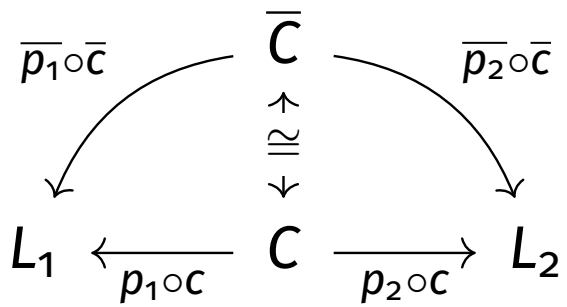

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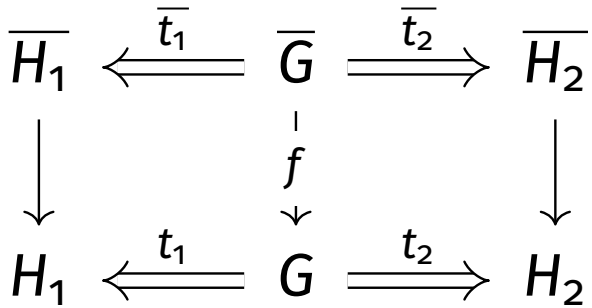


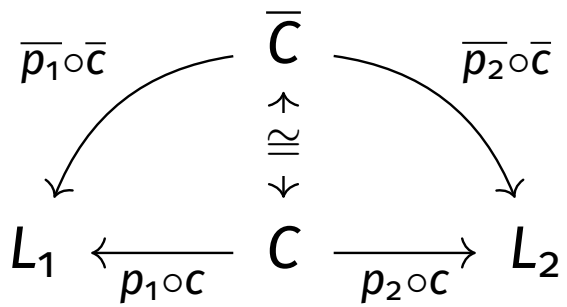

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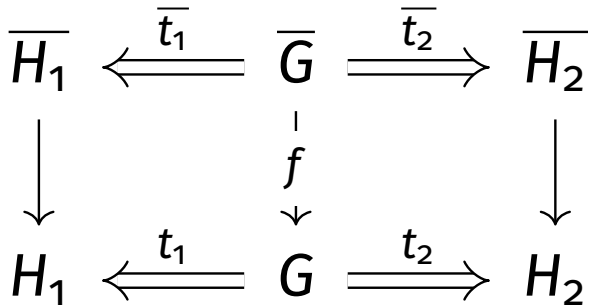


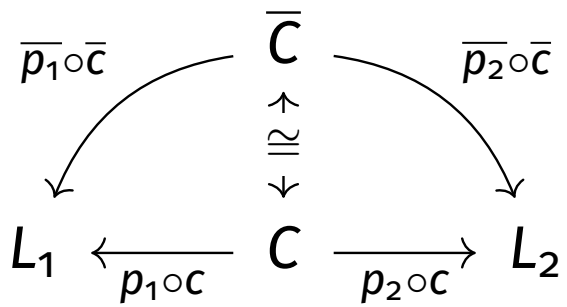


Conflicts are preserved and reflected by extension.

# Disabling Reasons

Essences are not the first proposed characterization

Given transformations $(t_1, t_2) : H_1 \overset{\rho_1, m_1}{\Longleftarrow} G \overset{\rho_2, m_2}{\Longrightarrow} H_2$

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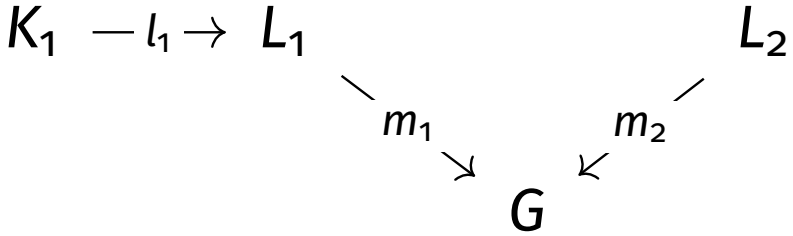

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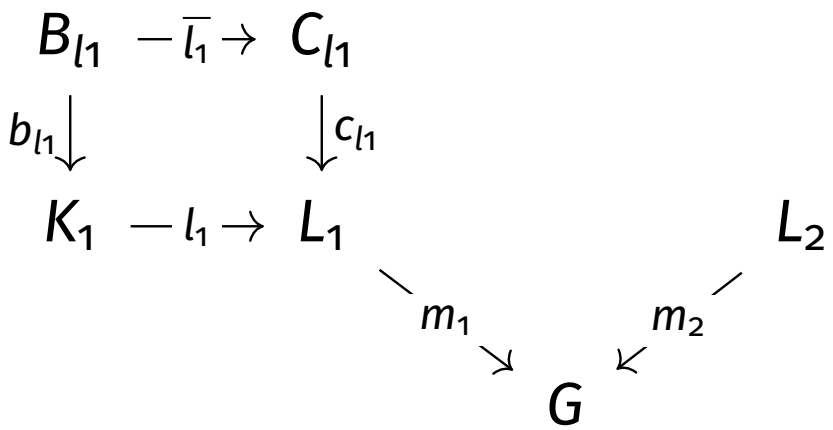

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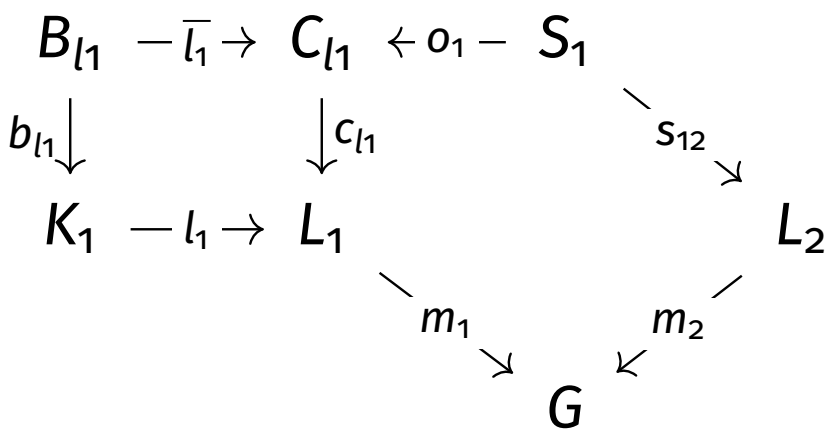

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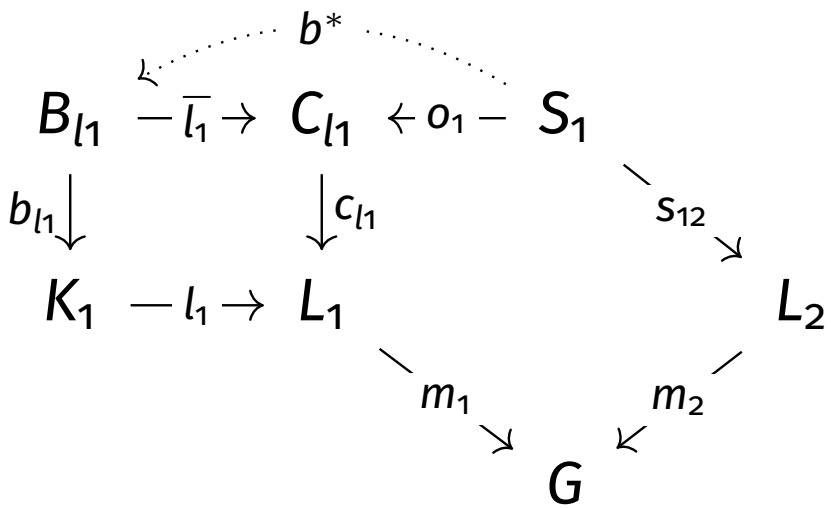

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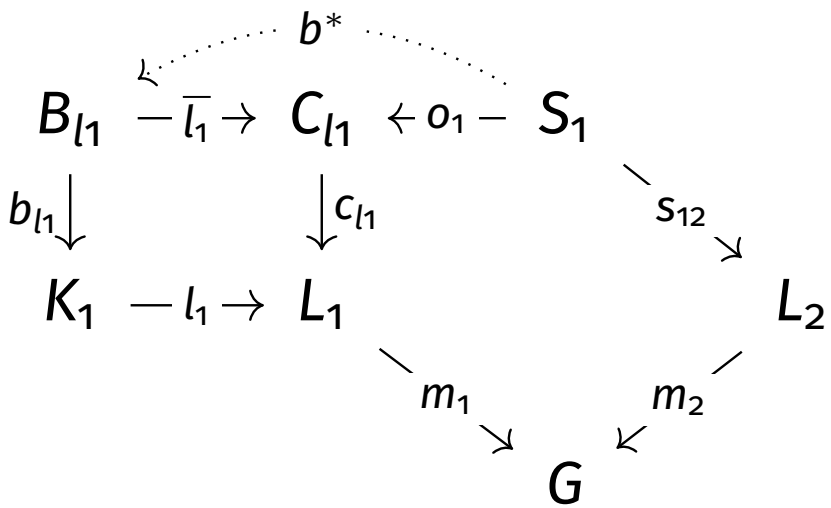
**Conflict reason** is union of *relevant* disabling reasons.



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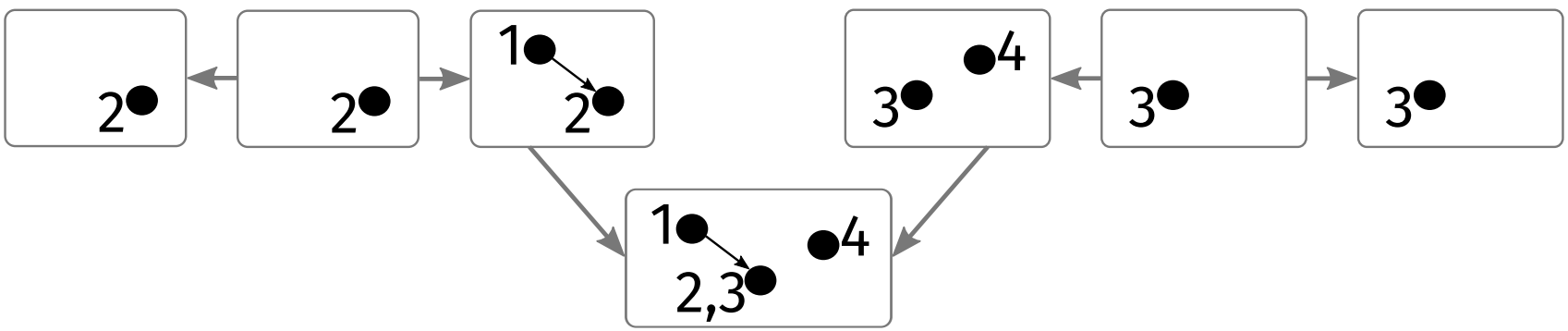

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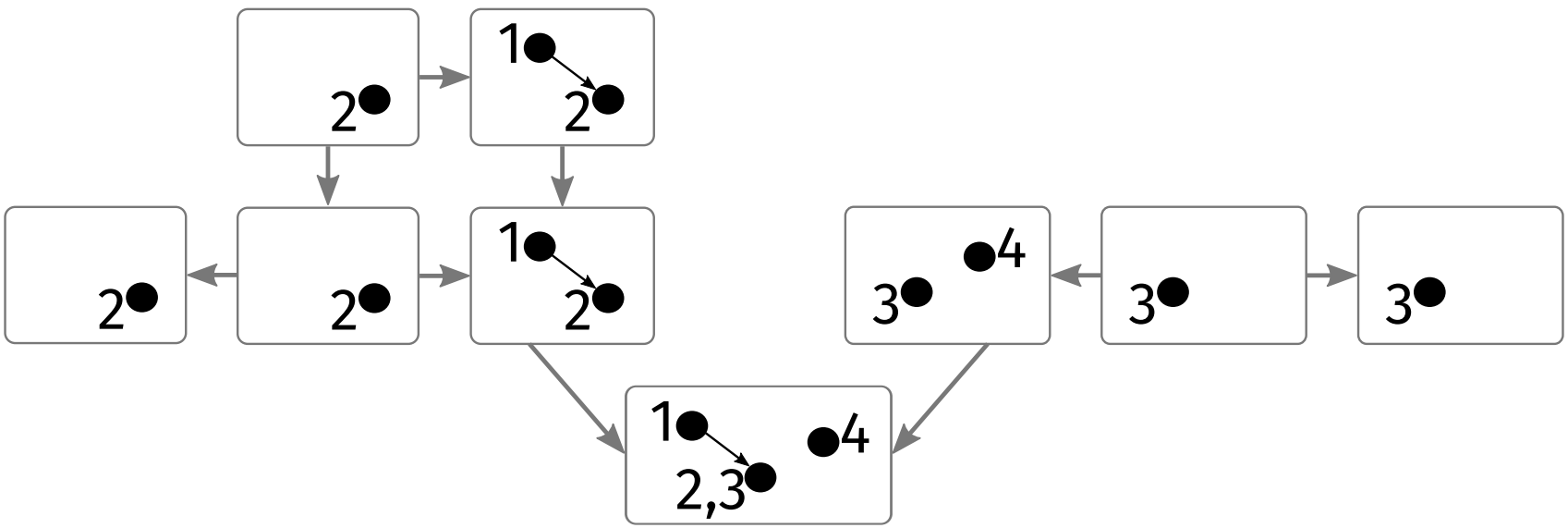

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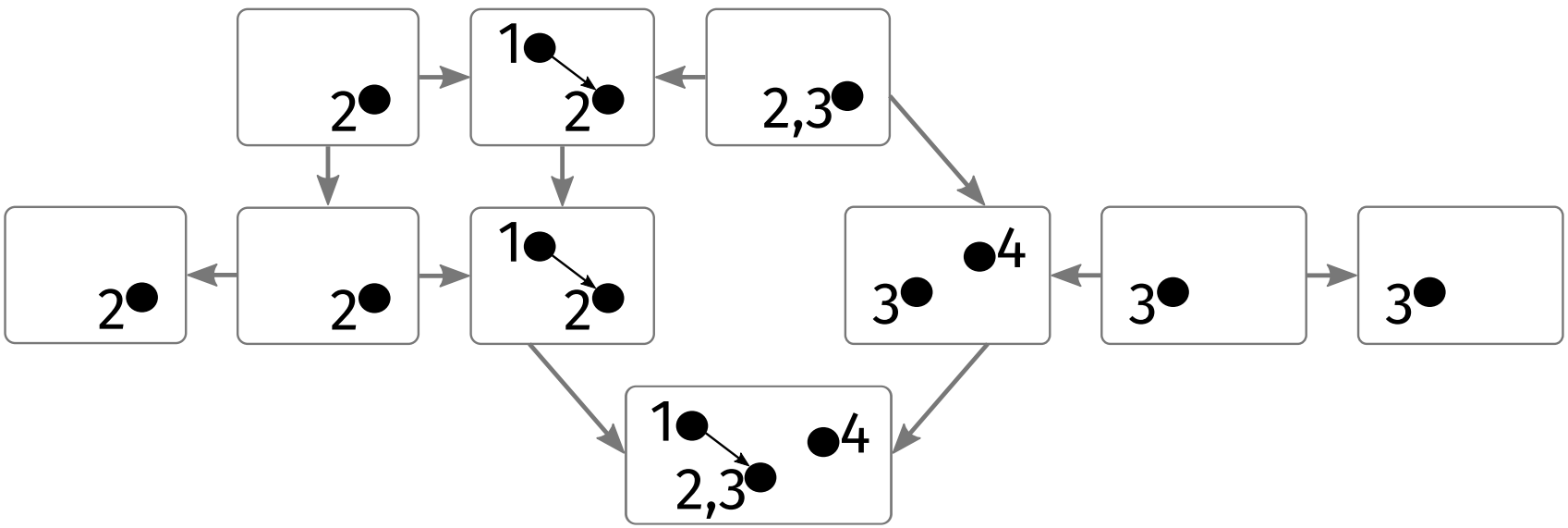

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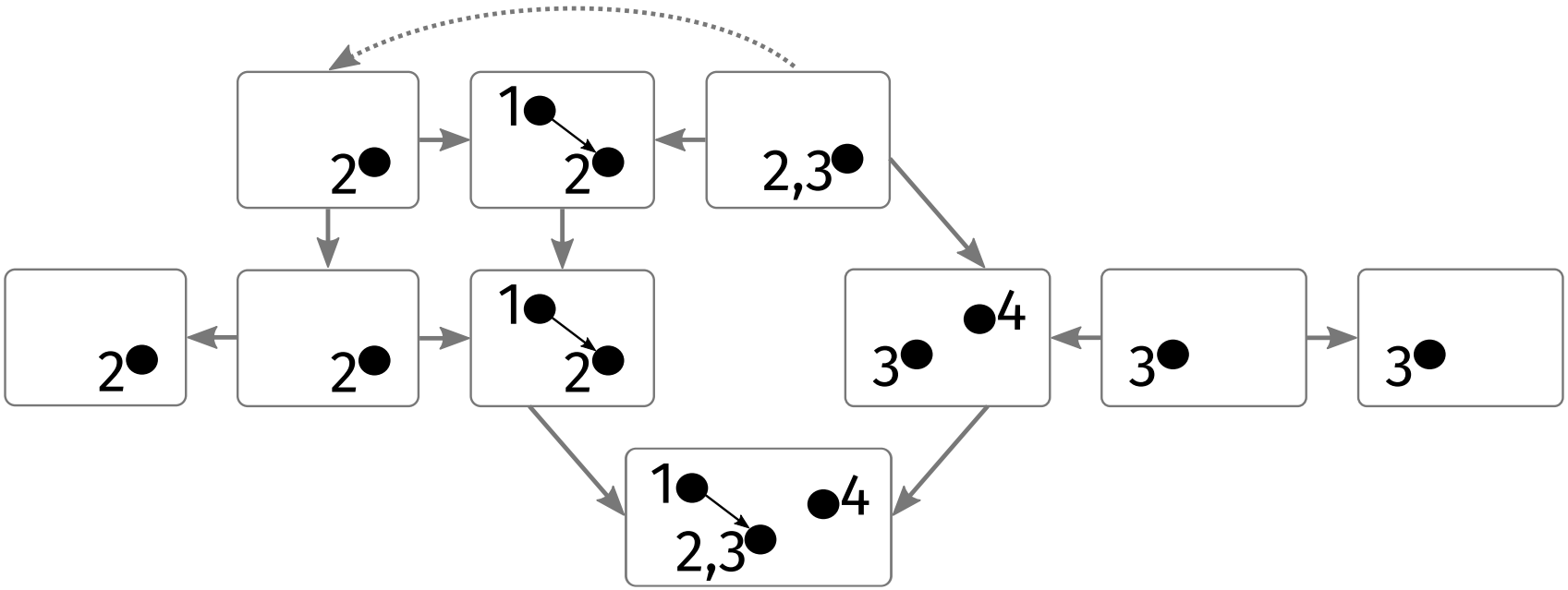

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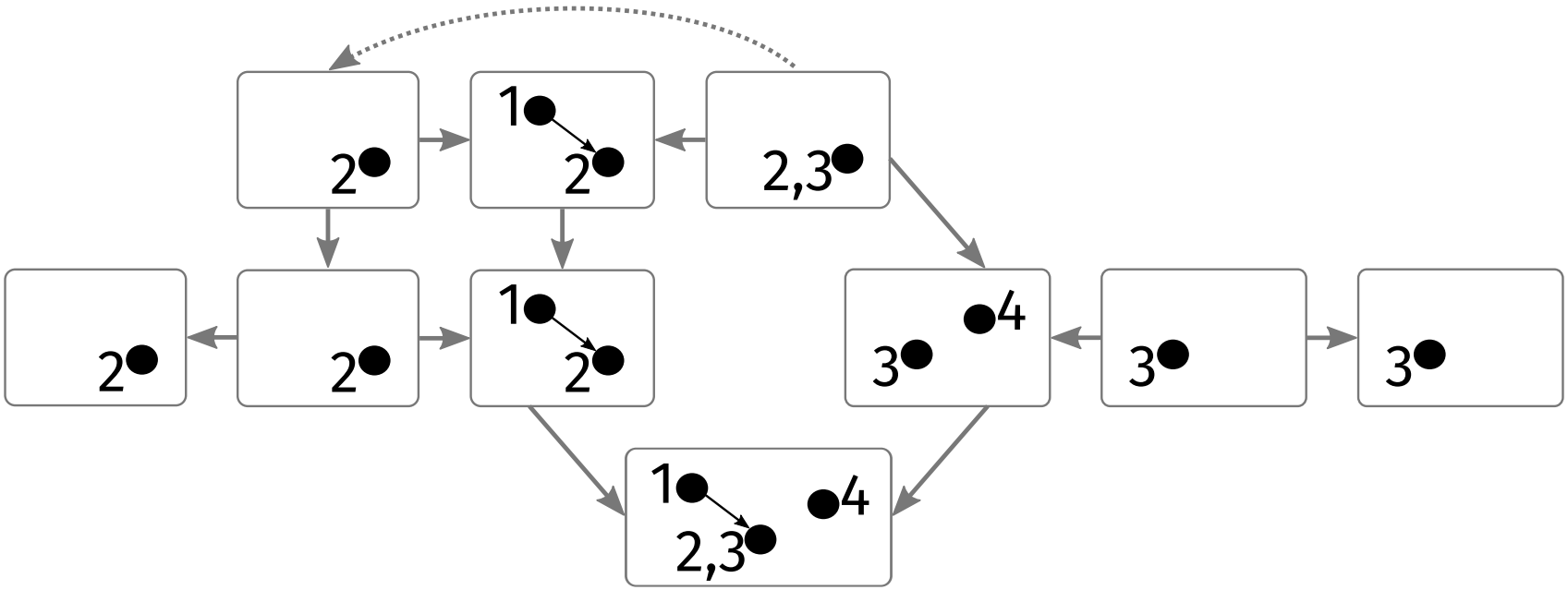


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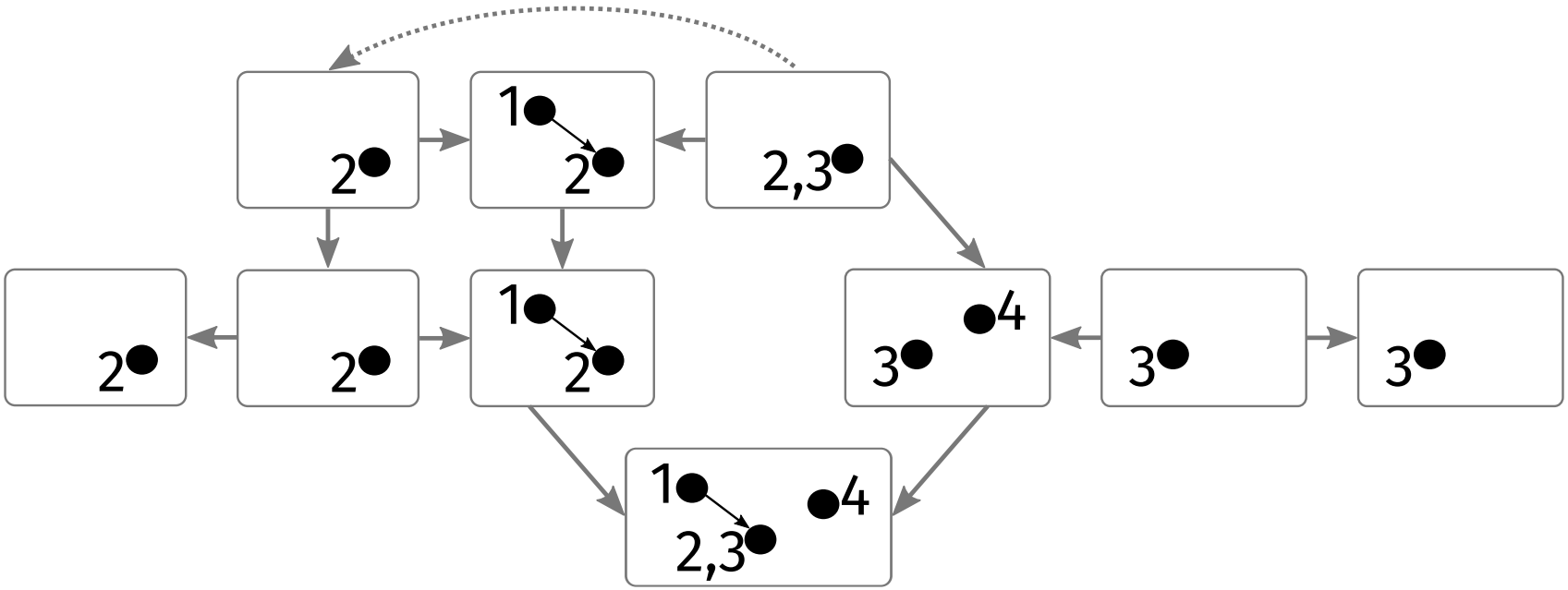


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- Inheritance also doesn't hold

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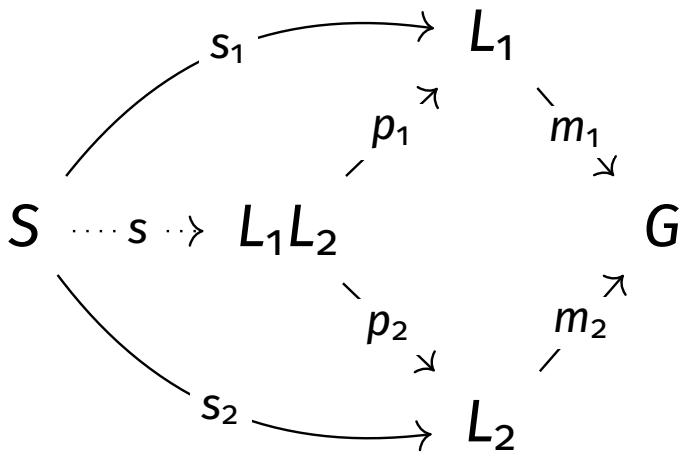
## Remark

Conflict reason determines $s \in \mathbf{Sub}(L_1 L_2)$.



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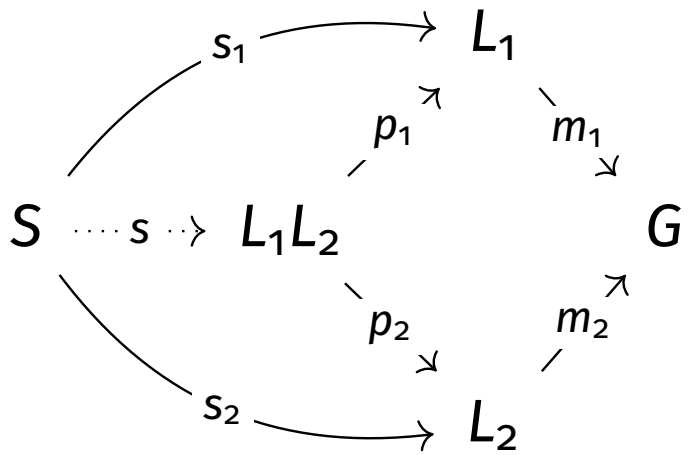


## Theorem

*If $c \in \mathbf{Sub}(L_1 L_2)$ is disabling essence and $s \in \mathbf{Sub}(L_1 L_2)$ disabling reason, then $c \subseteq s$.*
*The same holds if $c$ is conflict essence and $s$ conflict reason.*

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- We now understand **individual** conflicting transformations
- We want overview of **potential** conflicts for rules

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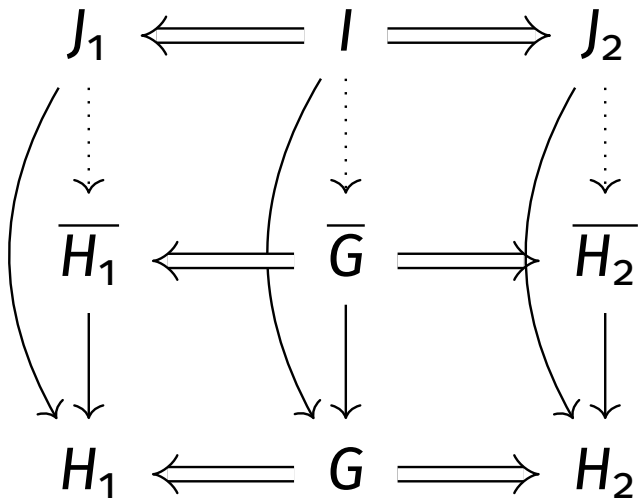
- Lambers, Born, et al. (2018) proposed **initial conflicts** (w.r.t extension)



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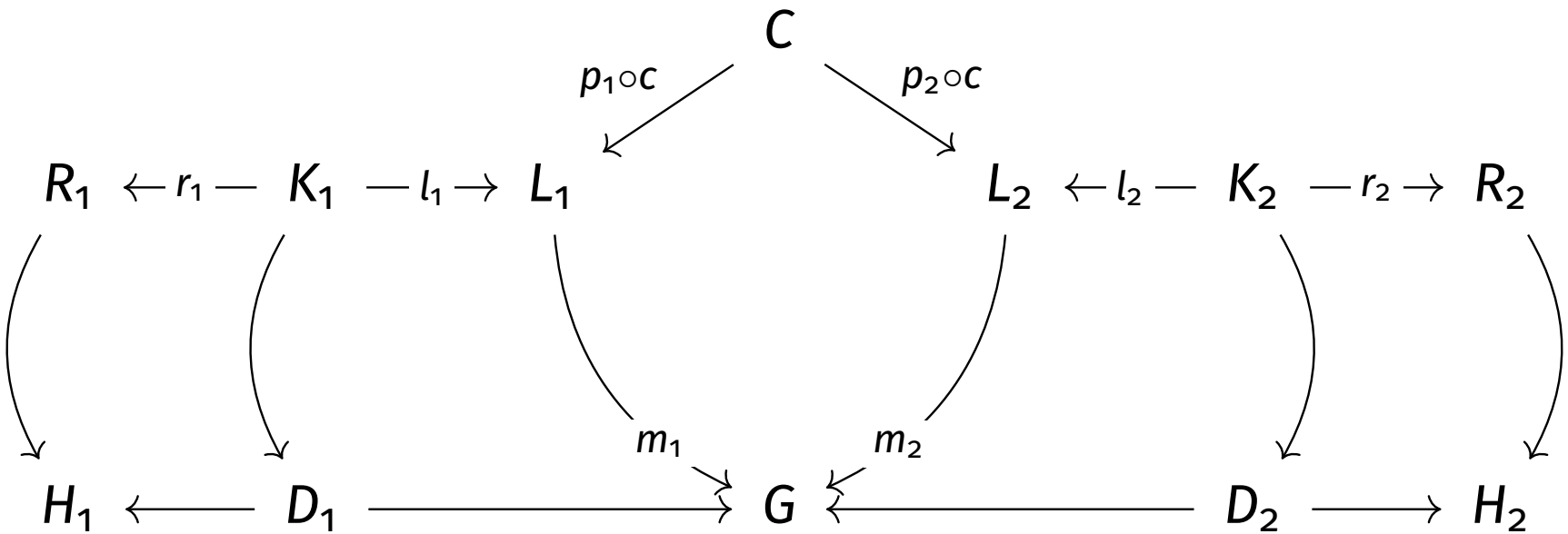

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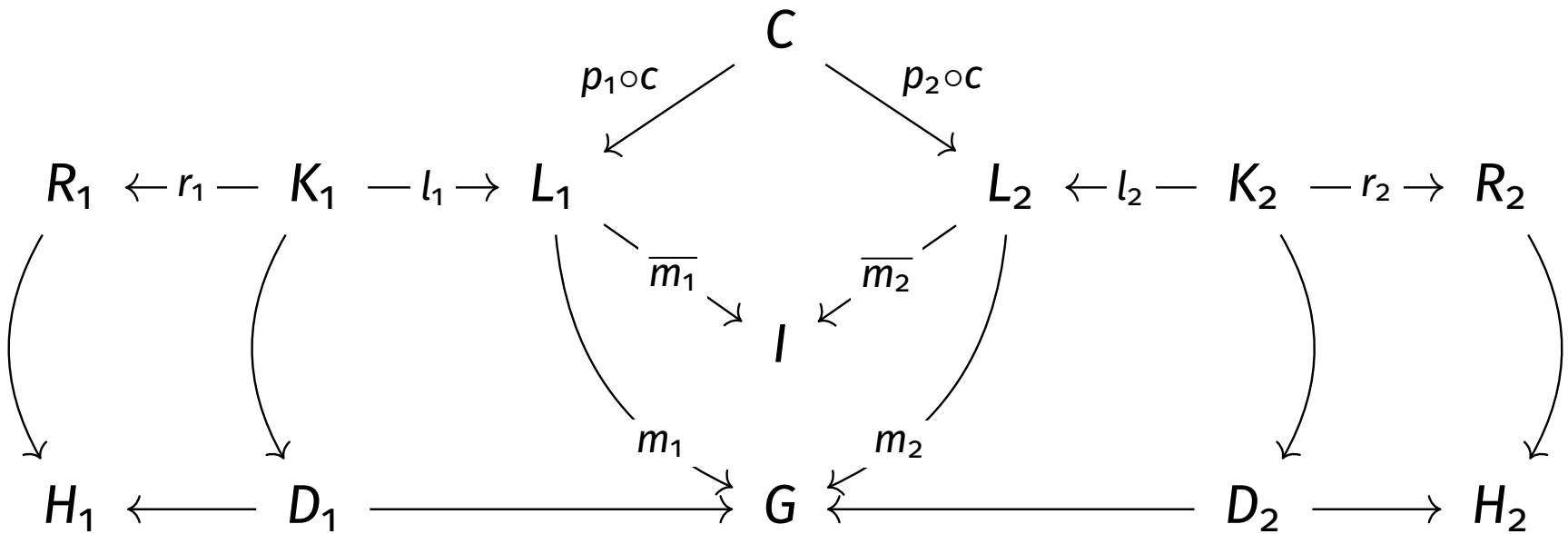

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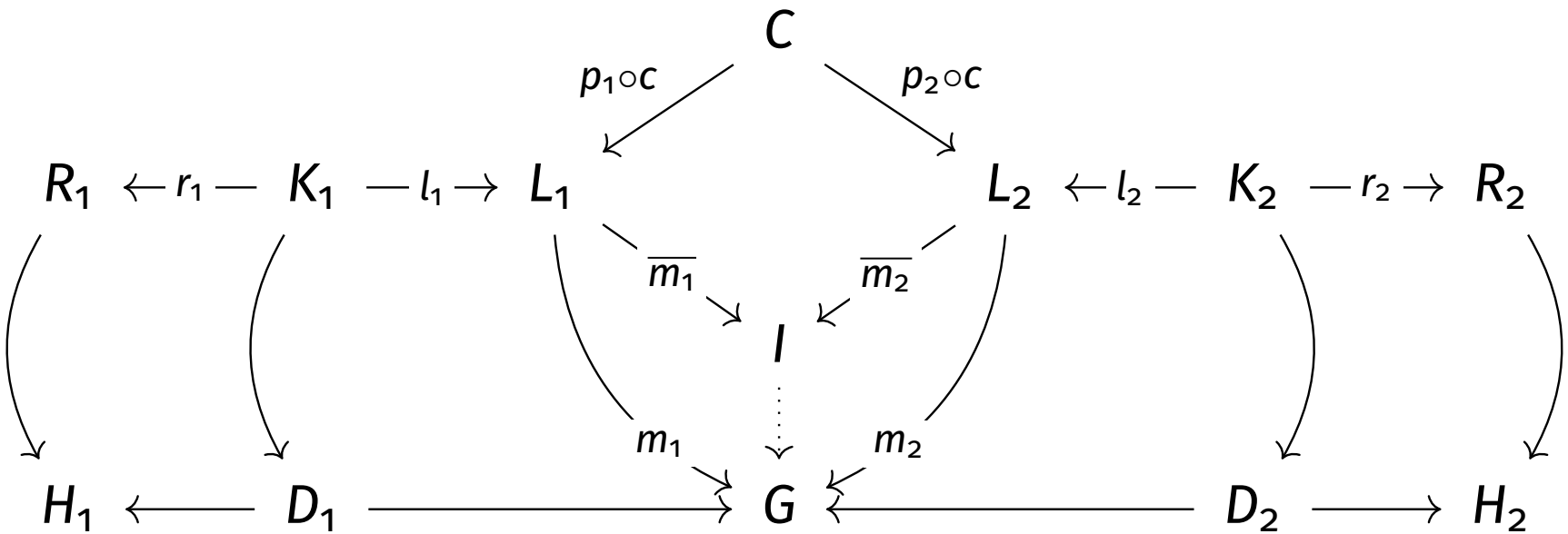

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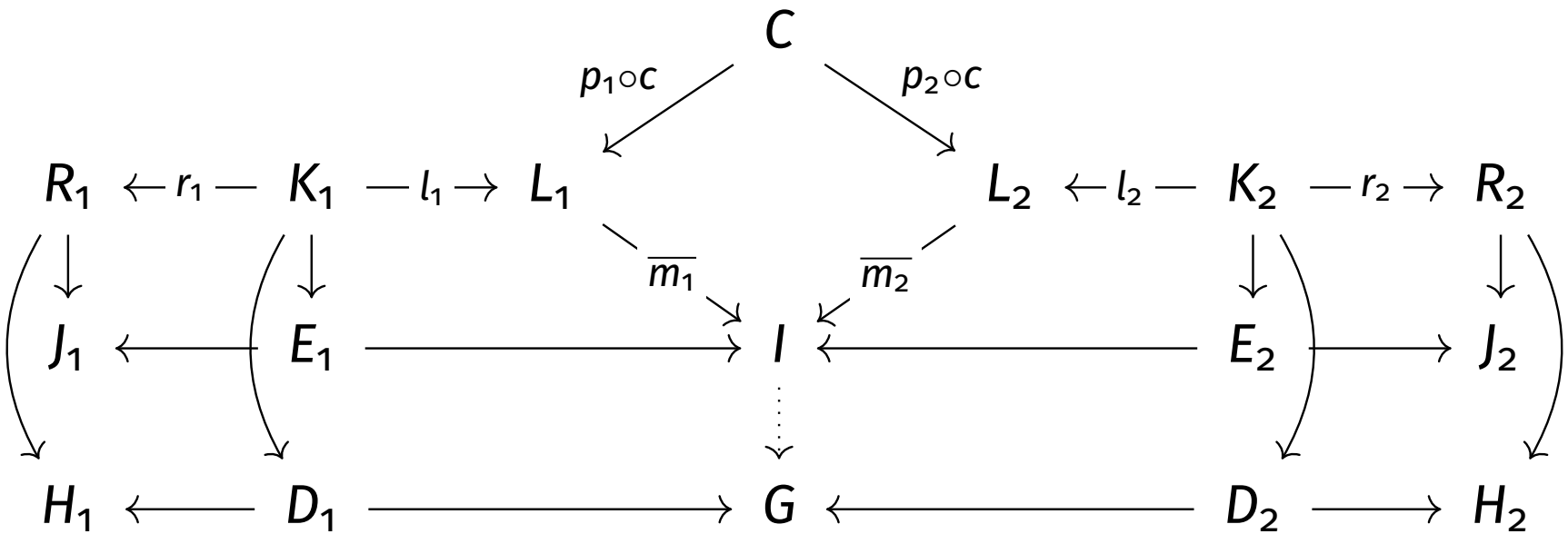

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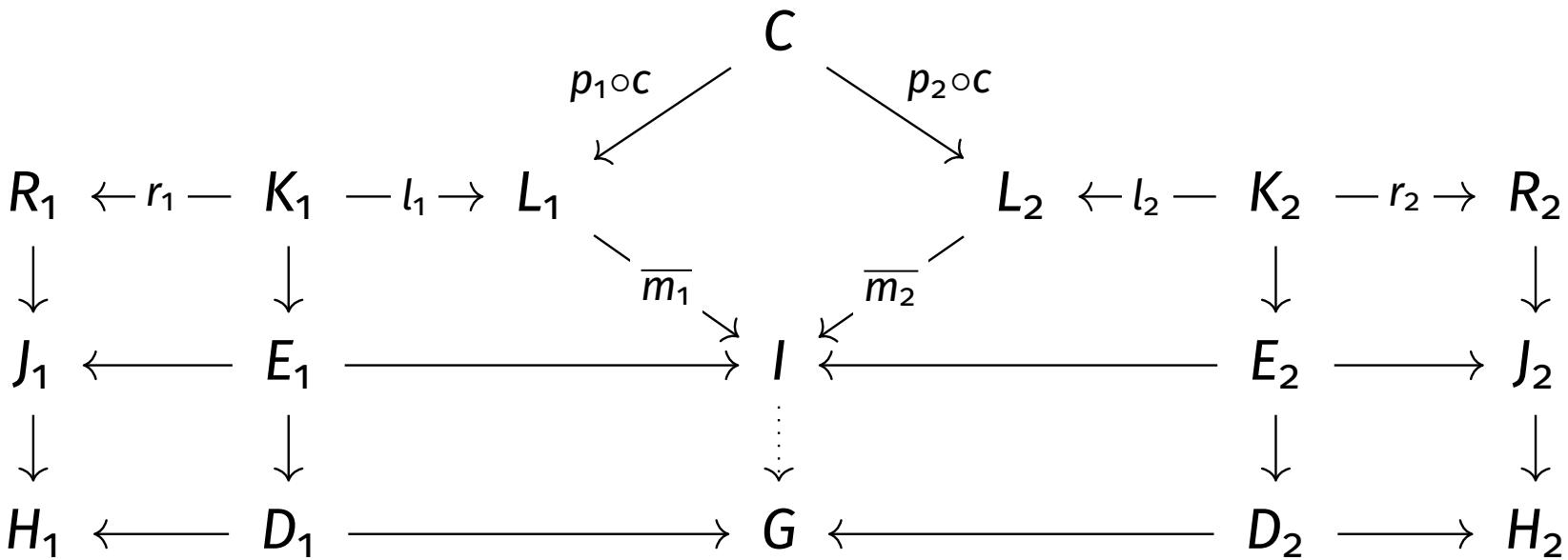

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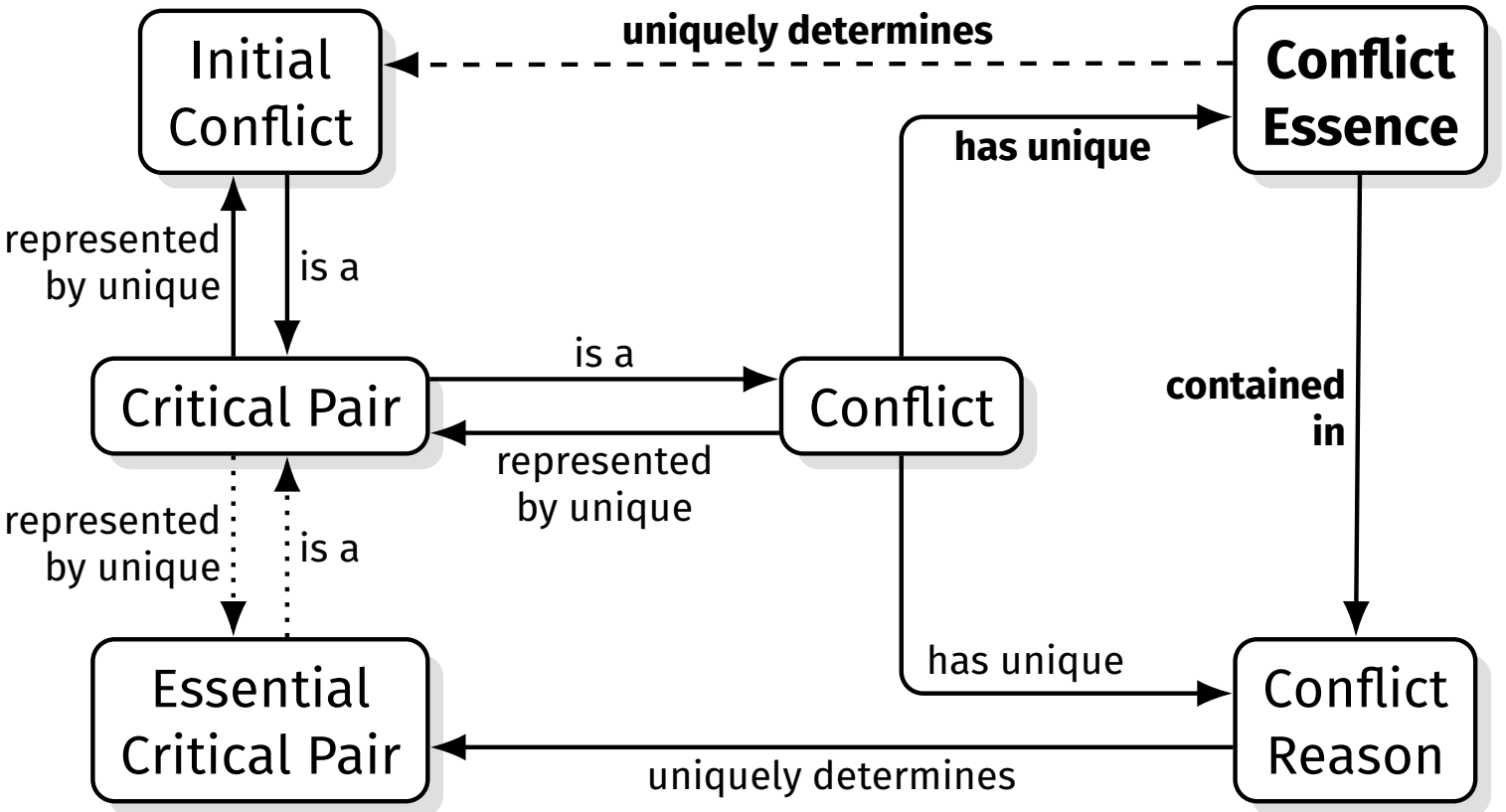
Available for: —— Adhesive Categories - - - - $\mathbb{S}\mathrm{et}^{\mathbb{S}}$ ····· $\mathbb{G}\mathrm{raph}_T$



# Overview

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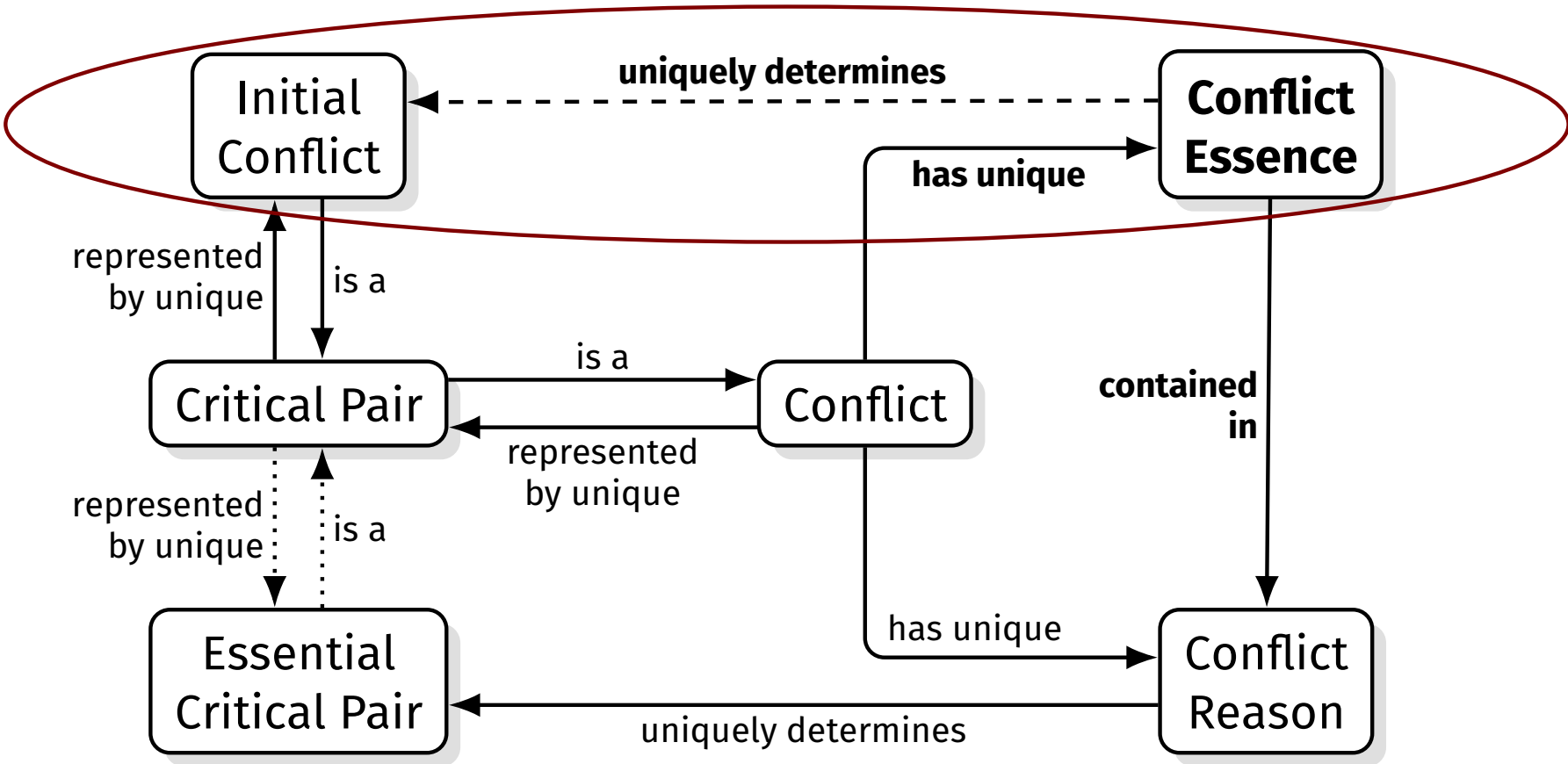

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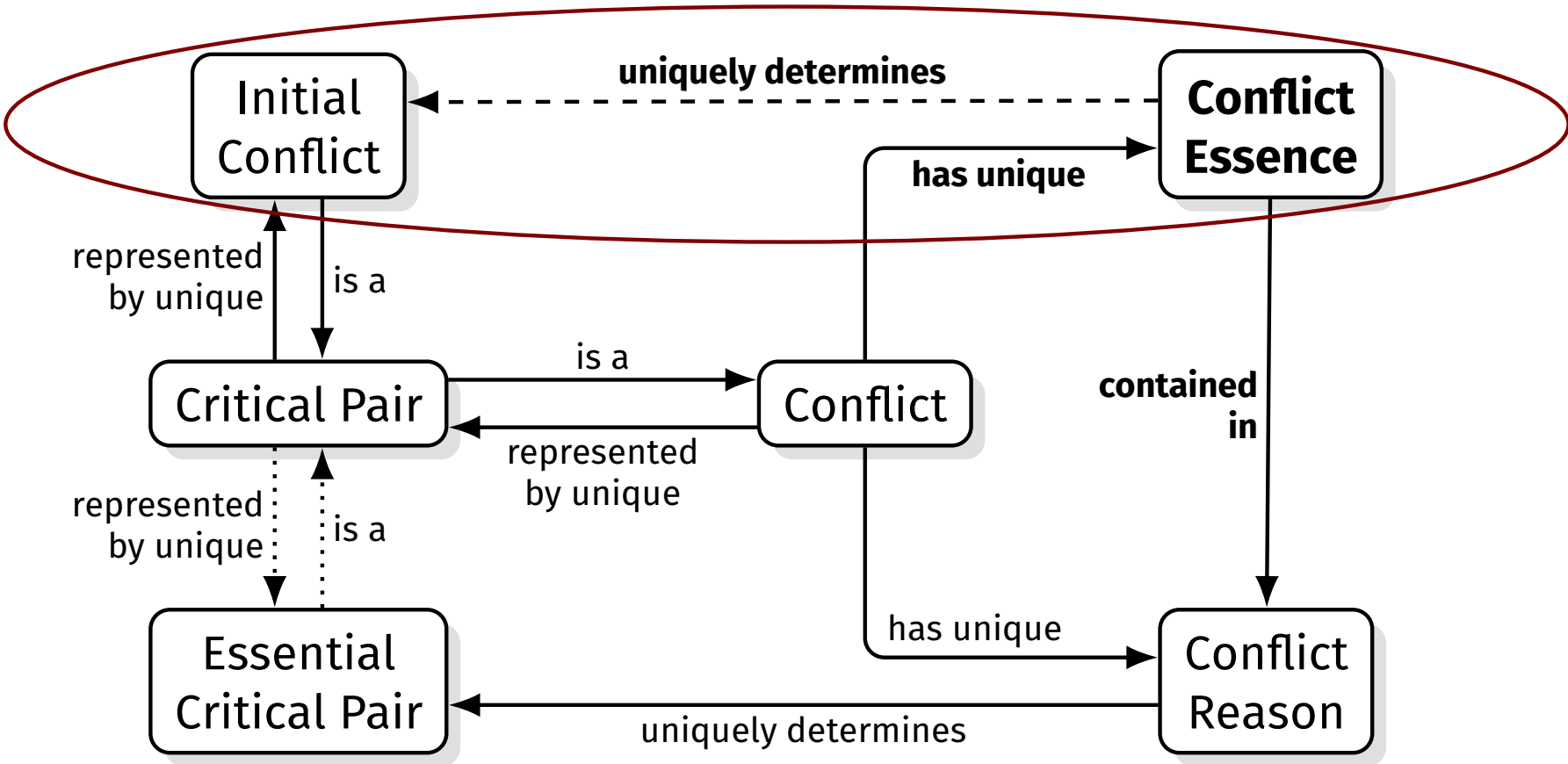


**Open Problem:** in all adhesive categories?

# Conclusions

- Essential condition allowed powerful characterization for root causes of conflicts
- Lots of future work!
  - Constraints and application conditions
  - Compare with notions of granularity (Born et al. 2017)
  - Attributed graphs and other adhesive categories
  - Sesqui-Pushout and AGREE

# Thank you!
## Questions?

# References I



Born, Kristopher et al. (2017). "Granularity of Conflicts and Dependencies in Graph Transformation Systems". In: *ICGT*. Vol. 10373. LNCS. Springer, pp. 125–141. DOI: 10.1007/978-3-319-61470-0_8. URL: https://doi.org/10.1007/978-3-319-61470-0_8.


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Lack, Stephen and Pawel Sobocinski (2005). "Adhesive and quasiadhesive categories". In: *ITA* 39.3, pp. 511–545. DOI: 10.1051/ita:2005028. URL: https://doi.org/10.1051/ita:2005028.

# References II


Lambers, Leen, Kristopher Born, et al. (2018). "Initial Conflicts and Dependencies: Critical Pairs Revisited". In: *Graph Transformation, Specifications, and Nets*. Vol. 10800. LNCS. Springer, pp. 105–123. DOI: 10.1007/978-3-319-75396-6_6. URL: https://doi.org/10.1007/978-3-319-75396-6_6.

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