UNIVERSITY OF LEICESTER

Morphisms and Transformations of Potentially Inconsistent Graphs

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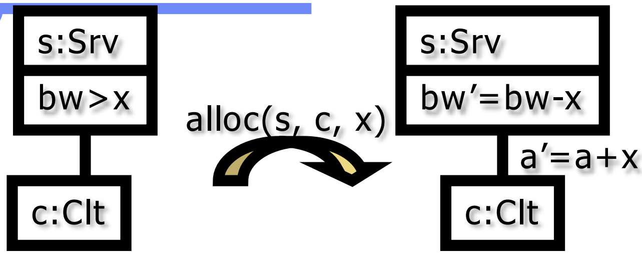
ROYAL HOLLOWAY UNIVERSITY OF LONDON

Ground Attributed Graph Transformation



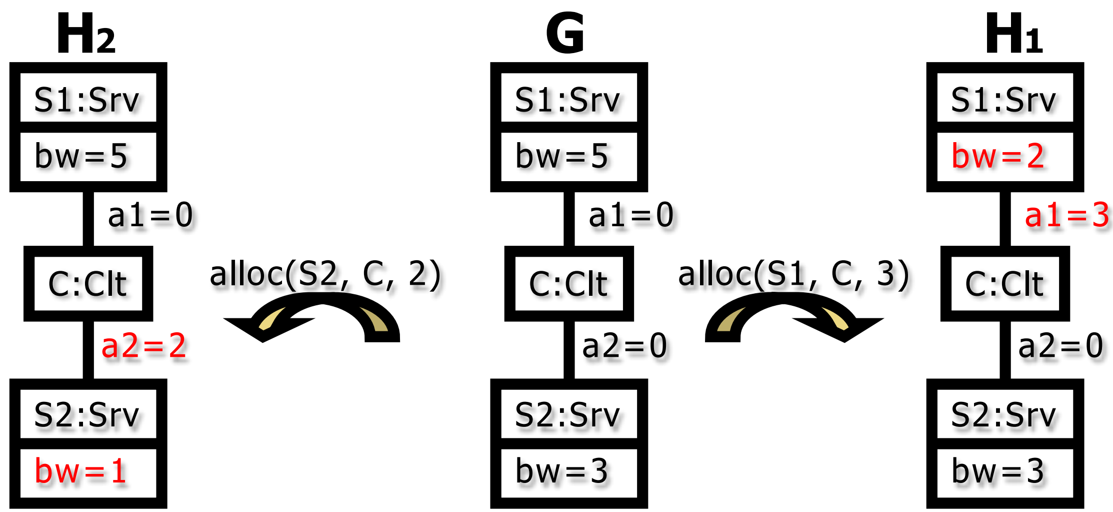
L, R attributed over **X** with constraints

- $\Phi_L = \Phi_R = \{s.bw>x,\ s.bw'=s.bw-x,\ e.a'=e.a+x\}$
- Match m: L → G satisfies $D \models \Phi_G \rightarrow m(\Phi_L)$

➔ Rule constraints satisfied by G's attributes

Ground symbolic graphs
SG = (G, Φ)

- G attributed over vars **X**
- constraints Φ = {x = d, …} with
  - vars x in **X**,
  - constants d in Σ-algebra **D**
- Invariant a1+a2<4

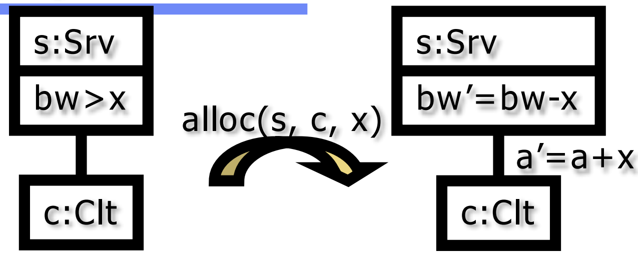


{S1.bw=5, a1=0, a2=2, S2.bw=1, a1+a2<4}

{S1.bw=5, a1=0, a2=0, S2.bw=3, a1+a2<4}

{S1.bw=2, a1=3, a2=0, S2.bw=3, a1+a2<4}

Symbolic Attributed Graph Transformation



L, R attributed over **X** with constraints

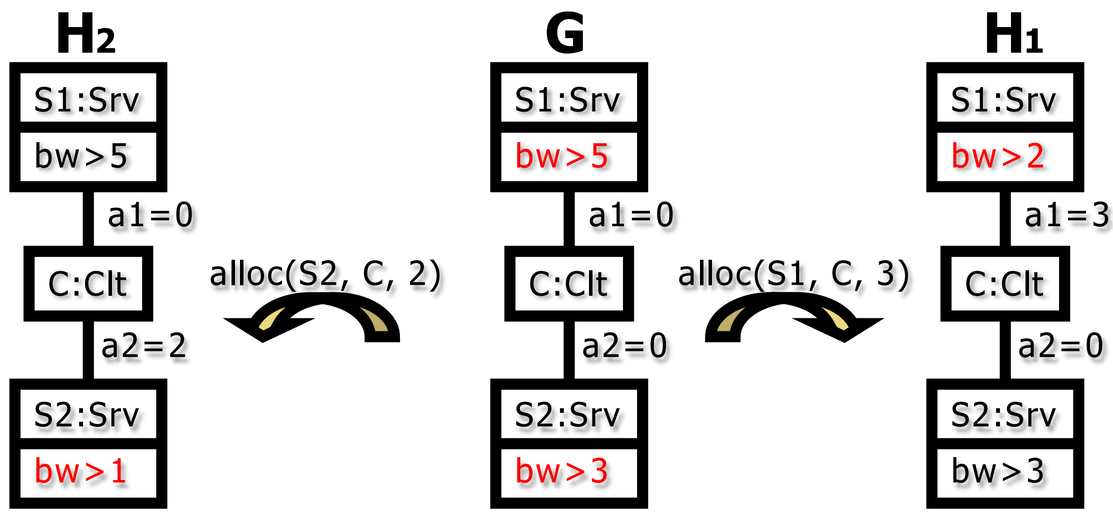
- $\Phi_L = \Phi_R = \{s.bw>x, s.bw'=s.bw-x, e.a'=e.a+x\}$
- Match m: L → G satisfies $D \models \Phi_G \rightarrow m(\Phi_L)$

➔ Rule constraints entailed by G's constraints

(General) symbolic graphs SG = (G, **Φ**)

- ✖ G attributed over vars X
- ✖ FO constraints **Φ** with
  - free vars in X and
  - constants from Σ-algebra D

Sem(SG) = set of attr. graphs satisfying **Φ**

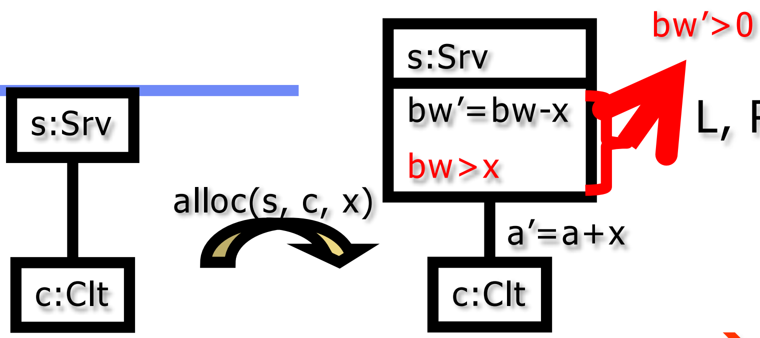


{S1.bw>5, a1=0, a2=2, S2.bw>1, a1+a2<4}

{S1.bw>5, a1=0, a2=0, S2.bw>3, a1+a2<4}

{S1.bw>2, a1=3, a2=0, S2.bw>3, a1+a2<4}

Symbolic Attributed Graph Transformation with Narrowing



L, R attributed over **X** with constraints

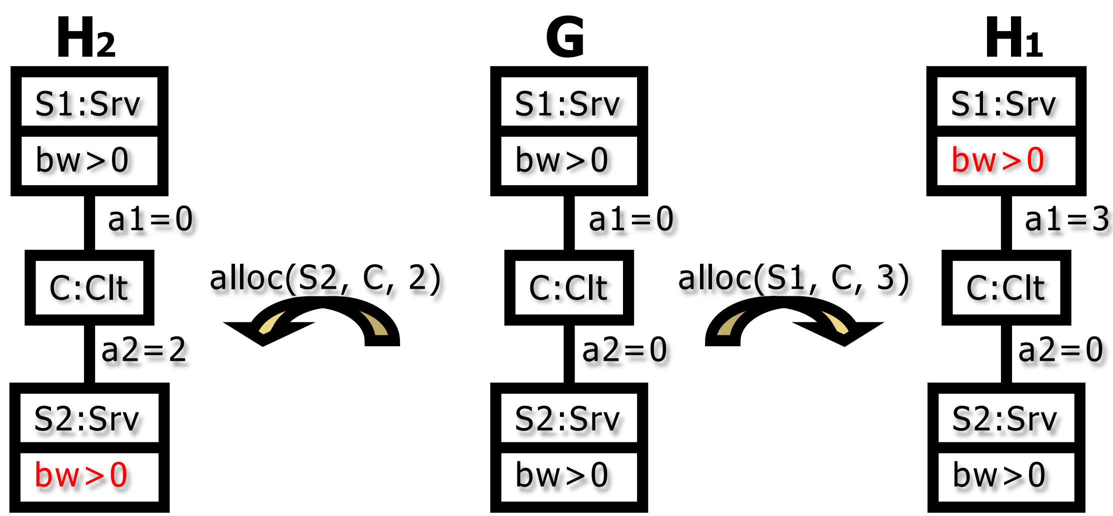
- $\Phi_L = \{\}$
- $\Phi_R = \{bw' = bw - x,\ bw' > 0,\ e.a' = e.a + x\} \cup \Phi_L$
- Match m: L → G satisfies $D \vDash \Phi_G \rightarrow m(\Phi_L)$
- Derived constraints $\Phi_G \cup m_i(\Phi_R)$ consistent

➔ L's constraints entailed by G's constraints

➔ R's constraints added to H, if consistent

(General) symbolic graphs SG = (G, **Φ**)

- ✖ G attributed over vars X
- ✖ FO constraints **Φ** with
  - free vars in X and
  - constants from **Σ**-algebra D



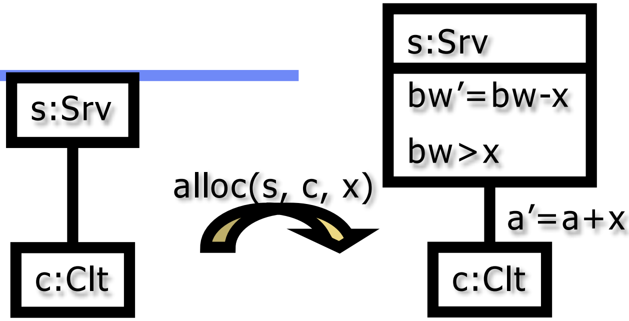
Sem(SG) = set of attr. graphs satisfying **Φ**

{S1.bw>0, a1=0, a2=2, S2.bw>0, a1+a2<4}

{S1.bw>0, a1=0, a2=0, S2.bw>0, a1+a2<4}

{S1.bw>0, a1=3, a2=0, S2.bw>0, a1+a2<4}

Symbolic Attributed Graph Transformation with Narrowing



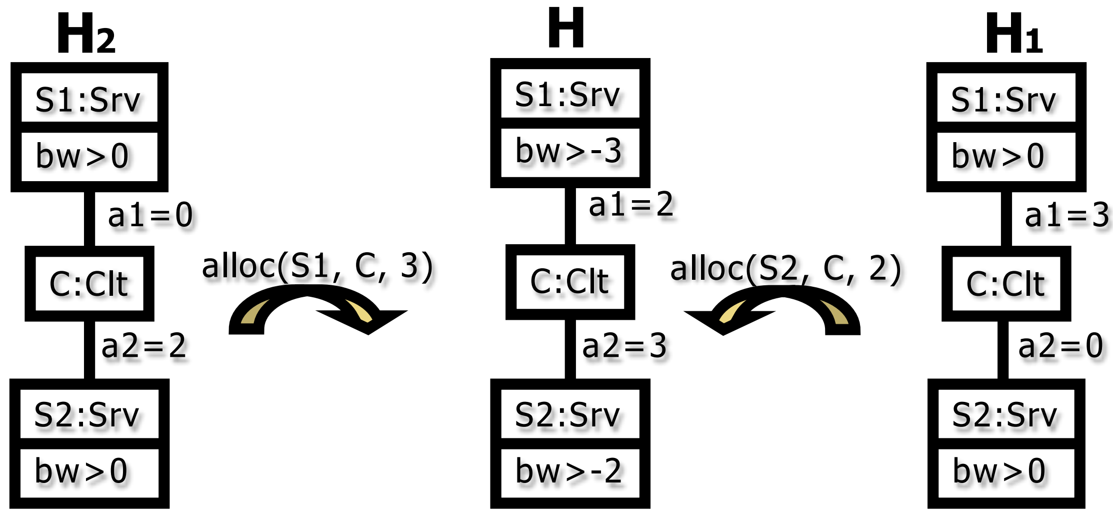
But: would like to

- decouple attribute handling from graph transformation, eg use external tool
- abstract representation of states with common graphs structure
- delay non-det. choices, retaining confluence

(General) symbolic graphs
SG = (G, Φ)

- G attributed over vars X
- FO constraints Φ with
  - free vars in X and
  - constants from Σ-algebra D

Sem(SG) = set of attr. graphs satisfying Φ

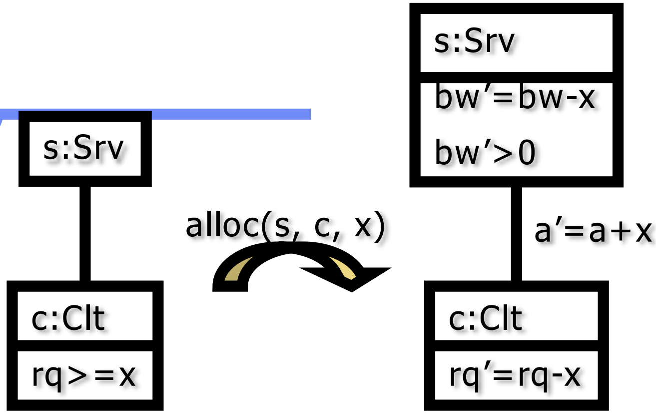


{S1.bw>0, a1=0, a2=2, S2.bw>0, a1+a2<4}

{S1.bw>-3, a1=2, a2=3, S2.bw>-2, a1+a2<4}

{S1.bw>0, a1=3, a2=0, S2.bw>0, a1+a2<4}

Potentially Inconsistent Graph Transformation



L, R attributed over **X** with constraints

- Φ_L , Φ_R as before
- Match m: L → G satisfies D ⊨ Ψ_G → m(Φ_L) for consistent subset Ψ_G of Φ_G

→ L's constraints entailed by cons. weak. of G's

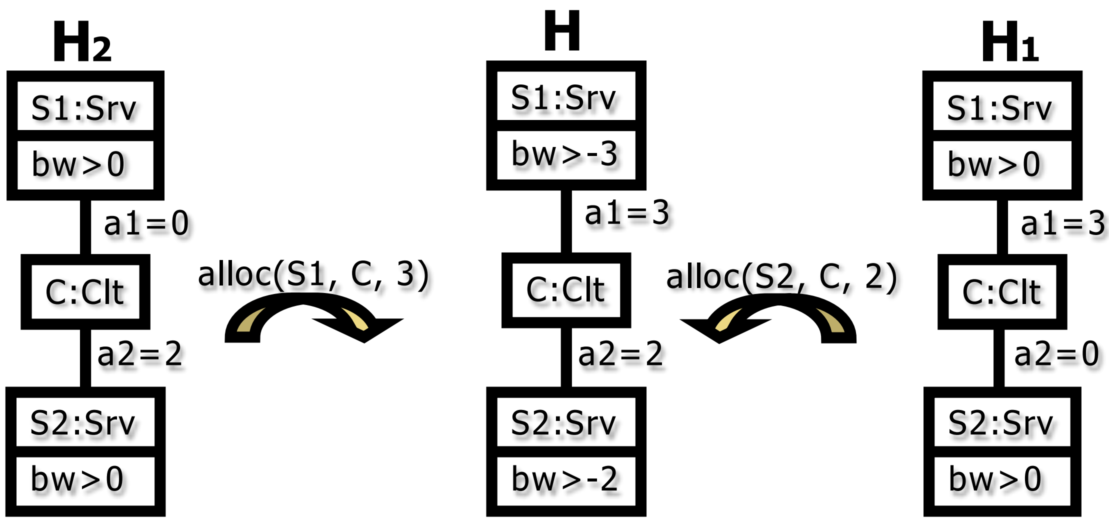
→ R's constraints added to H, pot. inconsistent

PIGs = symbolic graphs

PIG = (G, Φ)

Pot(PIG) = all (G, Ψ) s.t. Ψ is max consist subset of Φ

$$Sem(PIG) = \bigcup_{(G,\ \Psi)\ \text{in}\ Pot(PIG)} Sem(G, \Psi)$$



{S1.bw>0, a1=0, a2=2, S2.bw>0, a1+a2<4}

{S1.bw>-3, a1=2, a2=3, S2.bw>-2, a1+a2<4}

{S1.bw>0, a1=3, a2=0, S2.bw>0, a1+a2<4}

Any thoughts on …

- Consistent notion of deduction?
  - $\mathbf{\Phi} \vdash_c \varphi$ iff $\mathbf{\Phi} \vdash \varphi$ s.t. all formulas in the proof are consistent
  - See e.g. (Hunter & Nuseibeh 1998) on Inconsistency Management
- Institutions, categorical logic for para-consistency?
  - Is there a notion of morphism to get satisfaction condition, pushouts, etc?
- Precedents in state-based formal specifications, e.g., Z, B, …?

And now for the formal stuff…

- Morphisms
- Pushouts
- Institutions
- Local Church-Rosser