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Domains and Event Structures for Fusions

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Event Structures

- Concurrent semantics of a system in terms of
  - **Events** (~ computational steps)
  - **Dependencies** between events (enabling, causality, conflict etc.)

Reference Framework

[Nielsen, Plotkin, Winskel 78]

[Winskel 82]

equivalence of categories

Prime Event Structures $\underset{\longleftarrow}{\overset{\longrightarrow}{\simeq}}$ **Prime Domains**

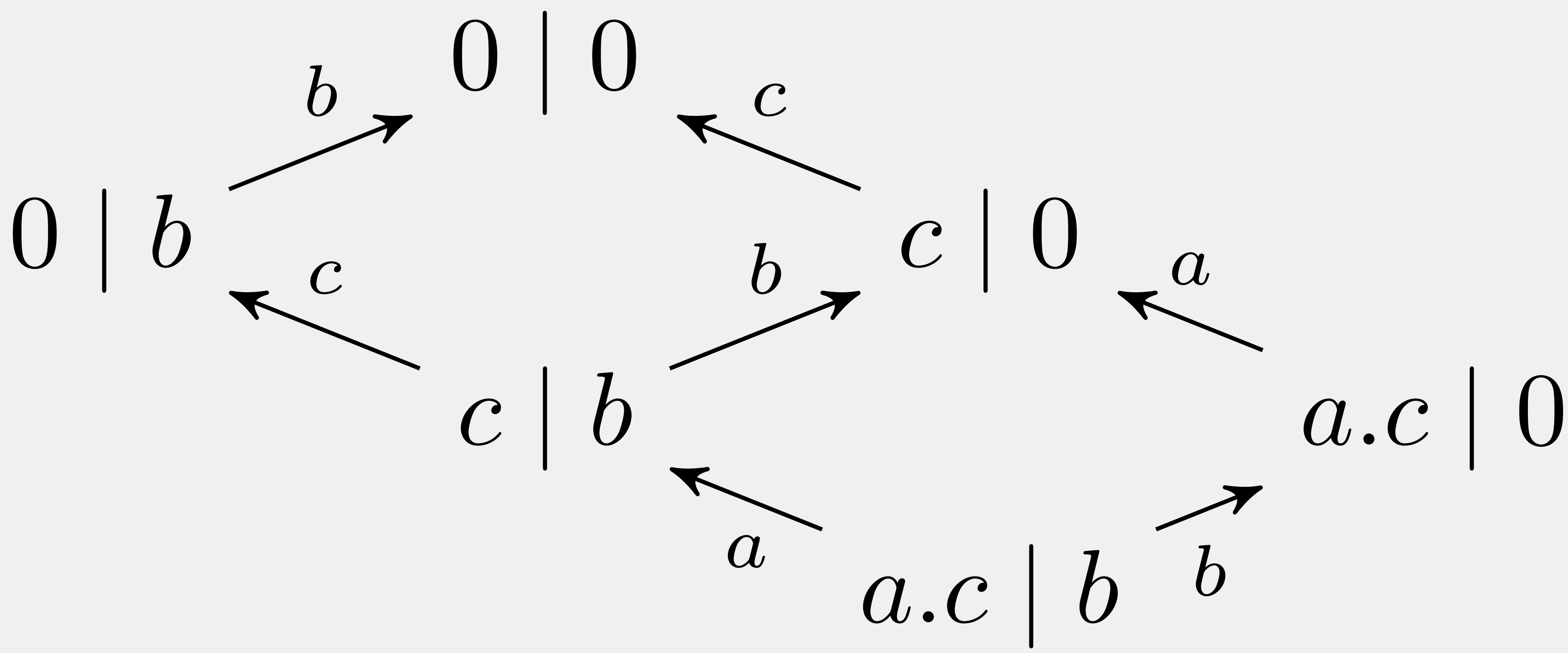
↑ ↓

Petri nets (CCS, …)

Partial order verification
[McMillan 92]

Example: CCS process

$a.c \mid b$



Example: CCS process

$a.c \mid b$

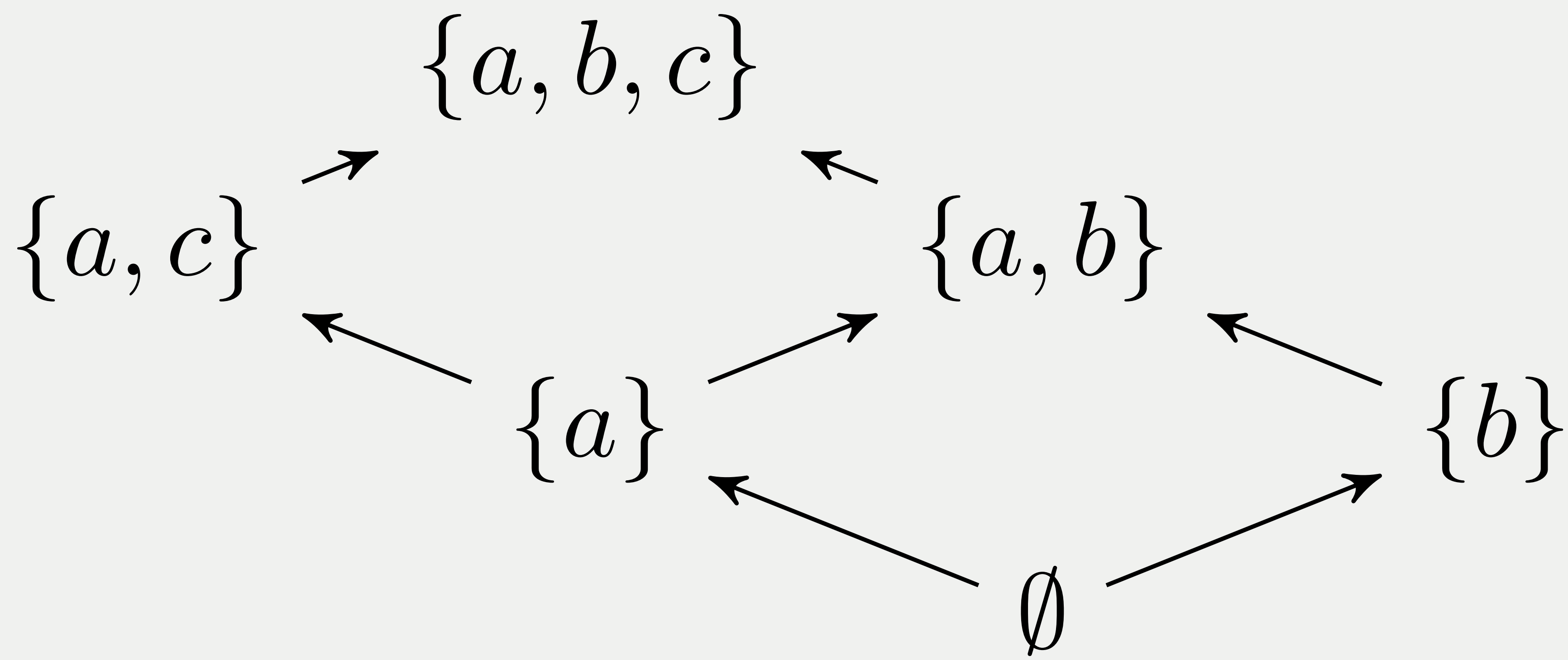
Events $a,\ b,\ c$

Enabling

$\emptyset \vdash a$

$\emptyset \vdash b$

$\{a\} \vdash c$



Configurations

Event Structures, in general

$\langle E, \vdash, \# \rangle$

E a set of events

$\vdash \subseteq 2^E_f \times E$ an enabling relation

$\# \subseteq E \times E$ a conflict relation

A *configuration* is a set of events C
- without conflicts
- events are "secured"

$C = \{e_1, e_2, e_3, \ldots\}$ $\qquad$ $\{e_1, \ldots, e_{k-1}\} \vdash e_k$

Prime Event Structures

Minimal enabling

$C \vdash e \quad$ s.t. $\quad \forall C' \subseteq C,\ C' \vdash e$ implies $C' = C$

Prime ES

each event has a **unique minimal enabling** set

(its causes)

$X \vdash e$ and $Y \vdash e$ imply $X \cap Y \vdash e$

Example

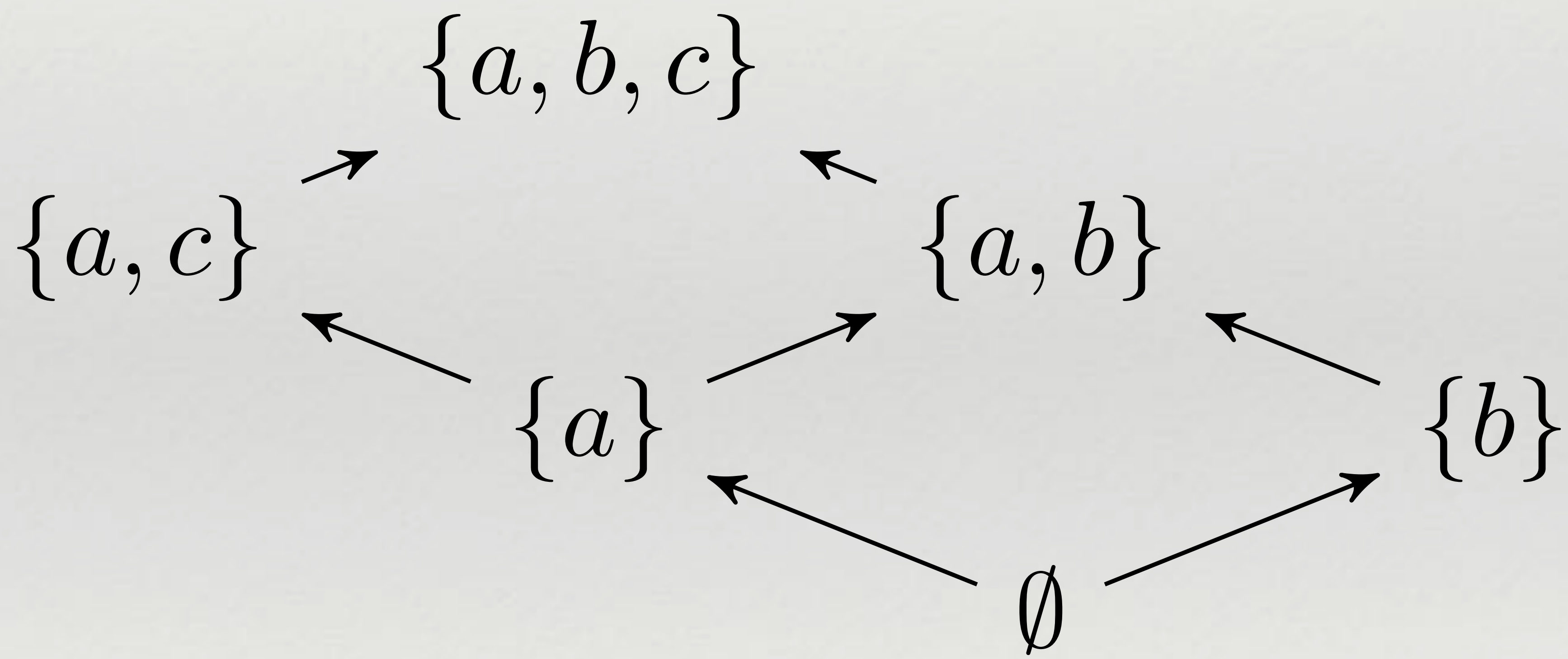
ES

$$\emptyset \vdash a$$

$$\emptyset \vdash b$$

$$\{a\} \vdash c$$

Configurations



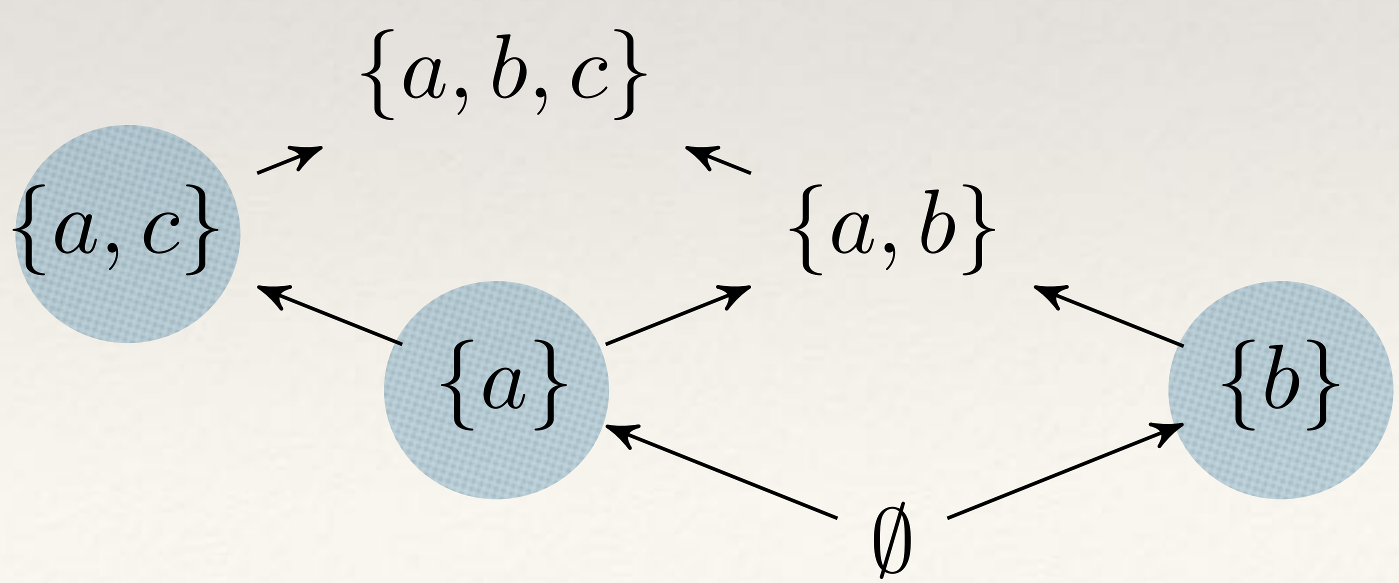
Prime Partial Orders

An element p of a poset is **prime** if

$$p \sqsubseteq \bigsqcup X \text{ then } \exists x \in X . p \sqsubseteq x$$

Prime partial order

each element is the join of the primes under it



Reference Framework, details

configurations
ordered by subset inclusion

Prime Event Structures $\underset{\longleftarrow}{\overset{\longrightarrow}{\cong}}$ **Prime Domains**

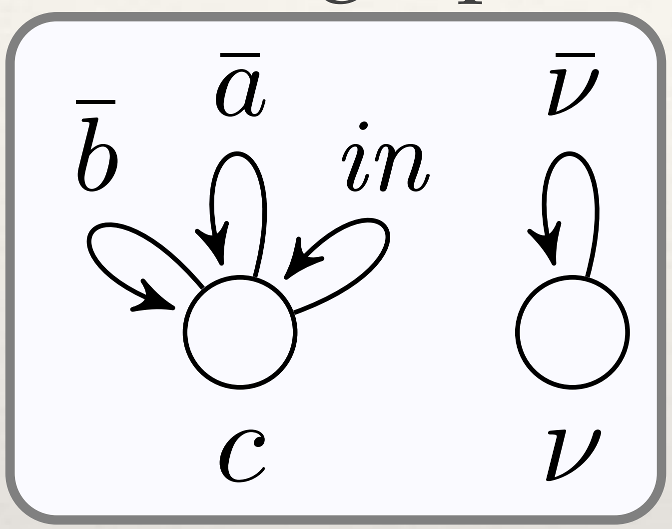
primes as events

coherent algebraic
finitary posets

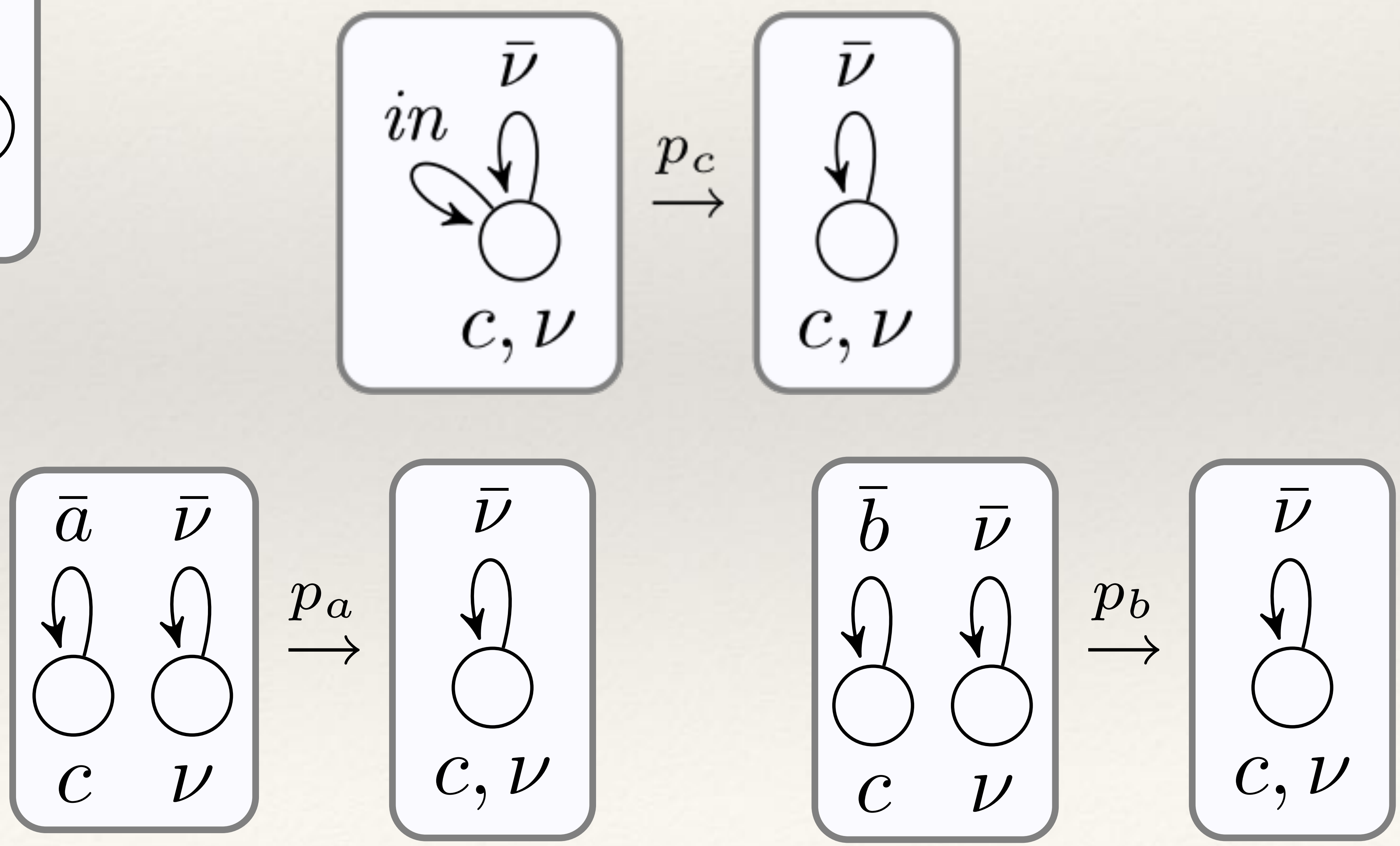
Introducing “fusions”

Graph Rewriting

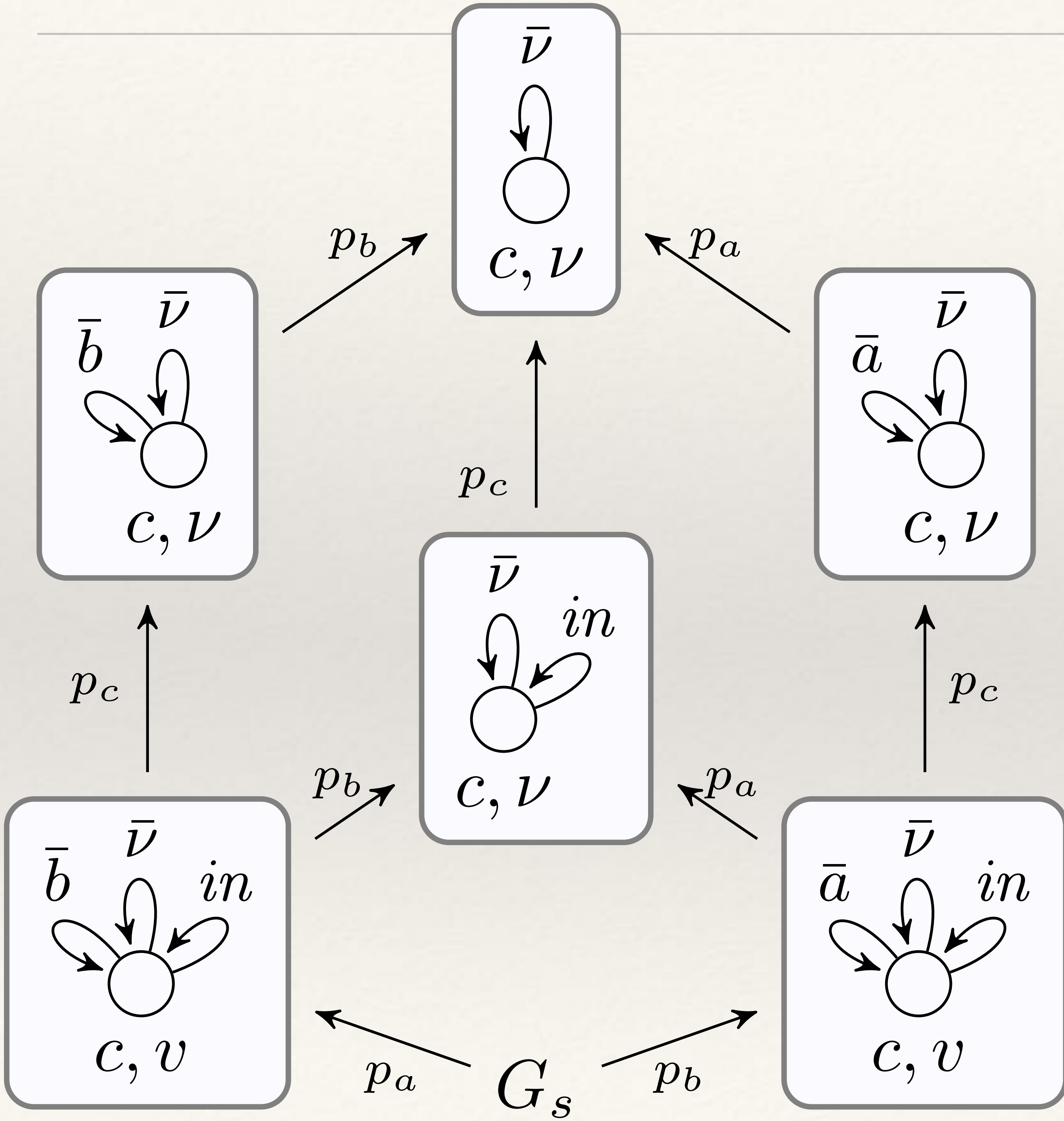
start graph



rules



the semantics



$\emptyset \vdash a$

$\emptyset \vdash b$

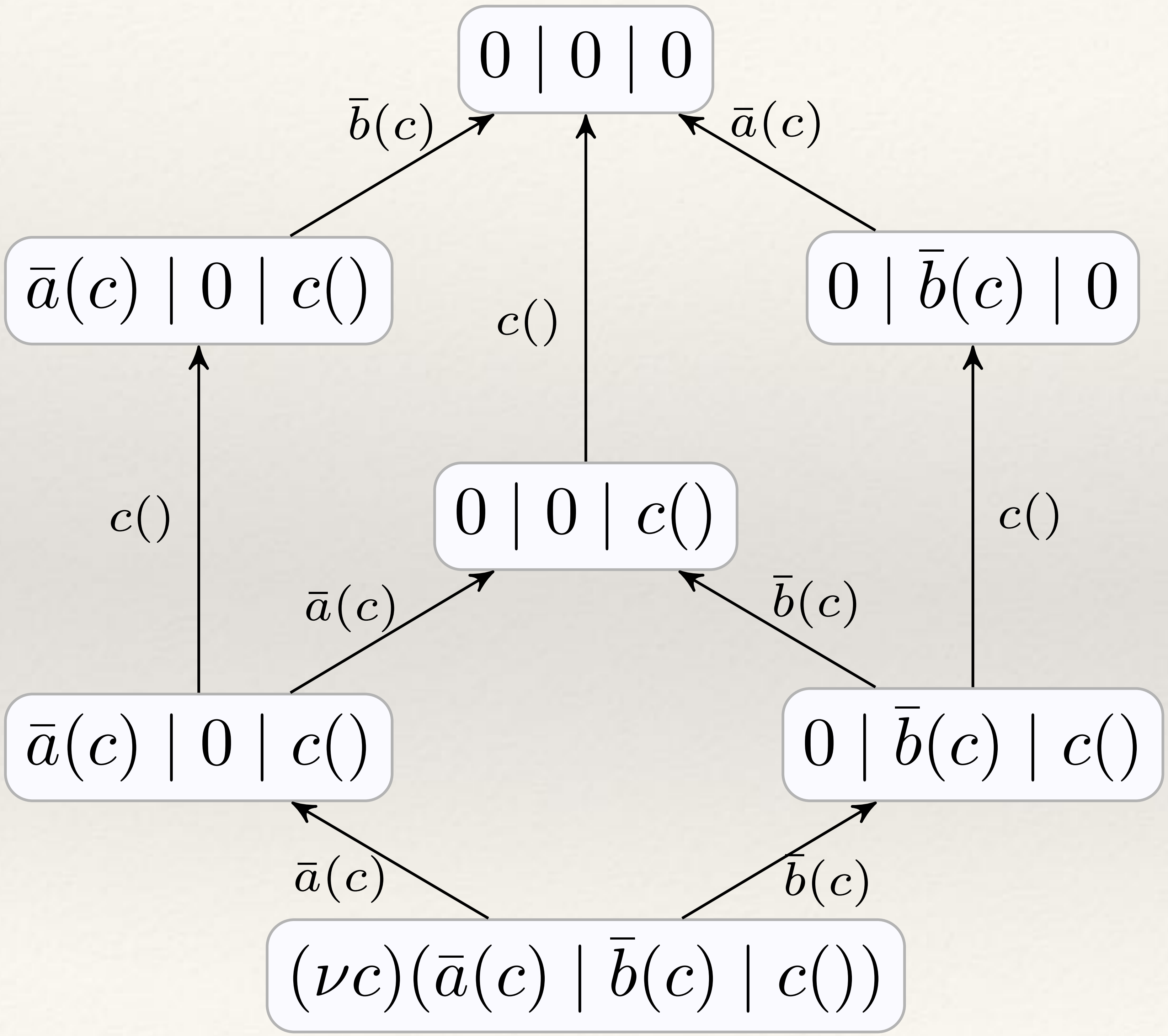
$\{a\} \vdash c$

$\{b\} \vdash c$

Not a prime ES!

(neither stable)

An eye to pi



$$\emptyset \vdash a$$

$$\emptyset \vdash b$$

$$\{a\} \vdash c$$

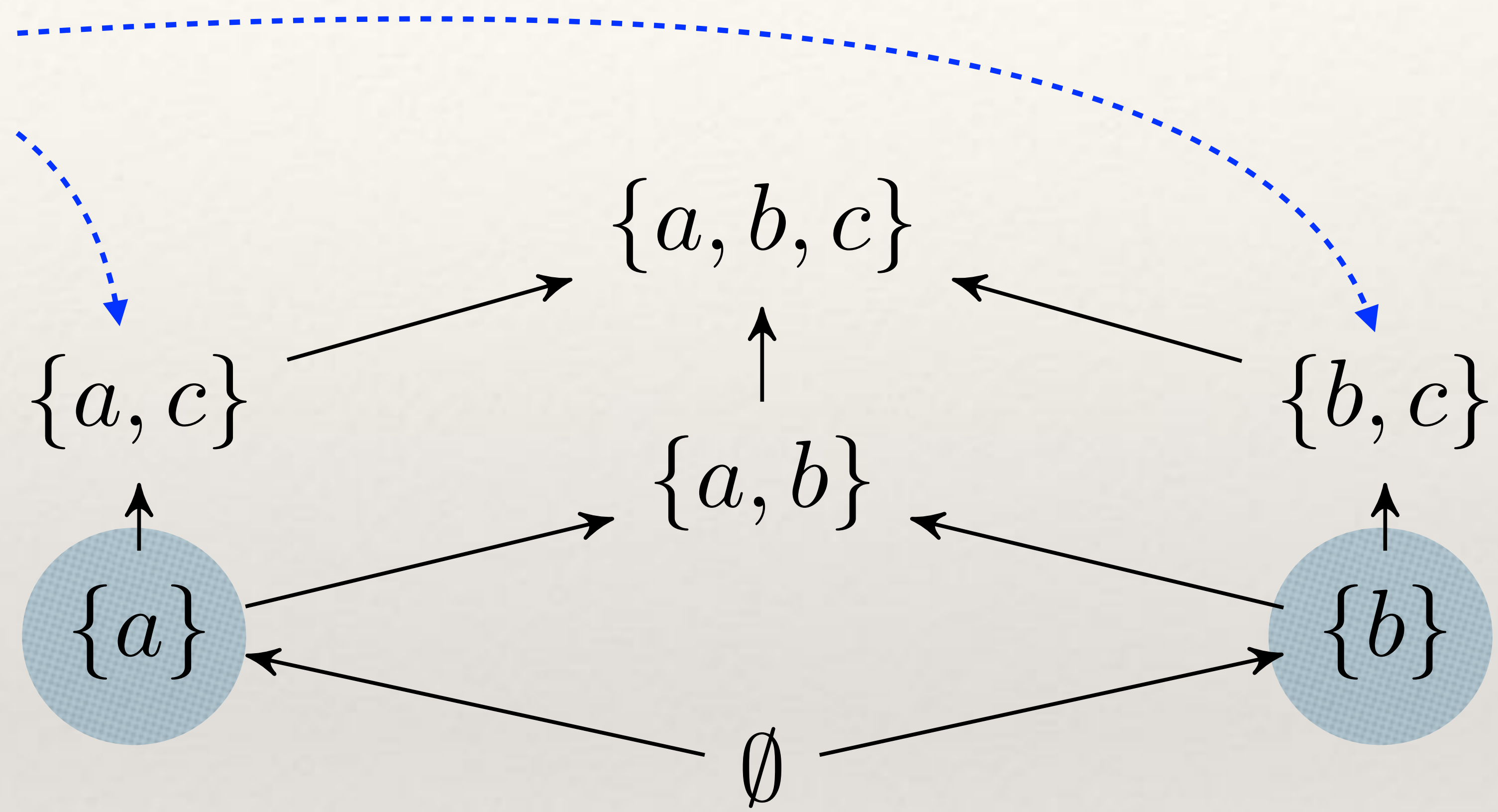
$$\{b\} \vdash c$$

Not a prime ES!

(neither stable)

Domain of Configurations

neither prime nor
join of primes

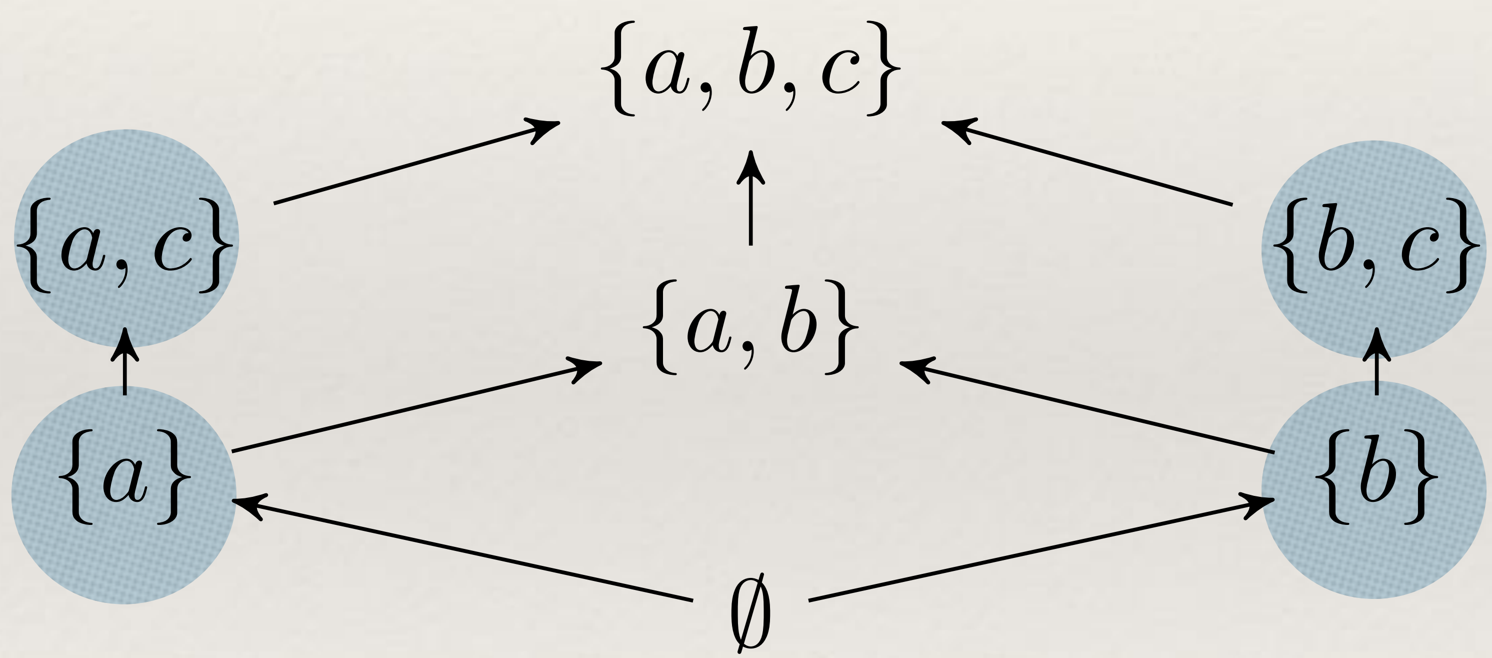


not a **prime** domain!

primes do not represent (all) "events"

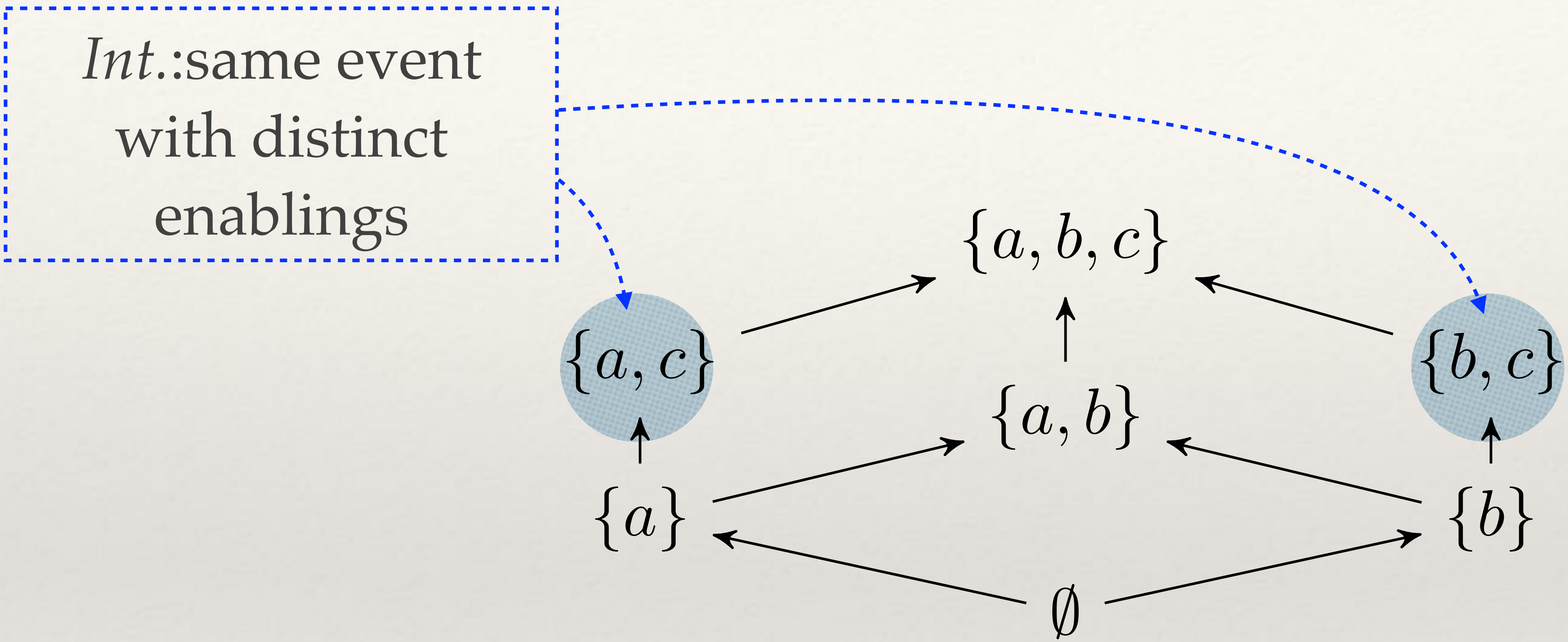
Irreducible Elements

An element *i* is **irreducible** if $i = \bigsqcup X$ then $i \in X$



[*i* is irreducible iff it has a unique predecessor *p(i)*]

Relating irreducibles



Formally: **interchangeable** in **join-decompositions**

$$\{a,b,c\} = \bigsqcup\{\{a\},\ \{b\},\ \{a,c\}\}$$
$$= \bigsqcup\{\{a\},\ \{b\},\ \{b,c\}\}$$

Interchange Relation

Interchangeable irreducibles

$i \leftrightarrow i'$ if $\bigsqcup(X \cup \{i\}) = \bigsqcup(X \cup \{i'\})$

for all decompositions

they can be used interchangeably in join-decompositions

Simpler Characterisation (in domains)

$i \leftrightarrow i'$ iff $p(i) \sqcup i' = p(i') \sqcup i$

the reference framework
generalises working with irreducibles
"up to interchangeability"

Weak Prime Partial Orders

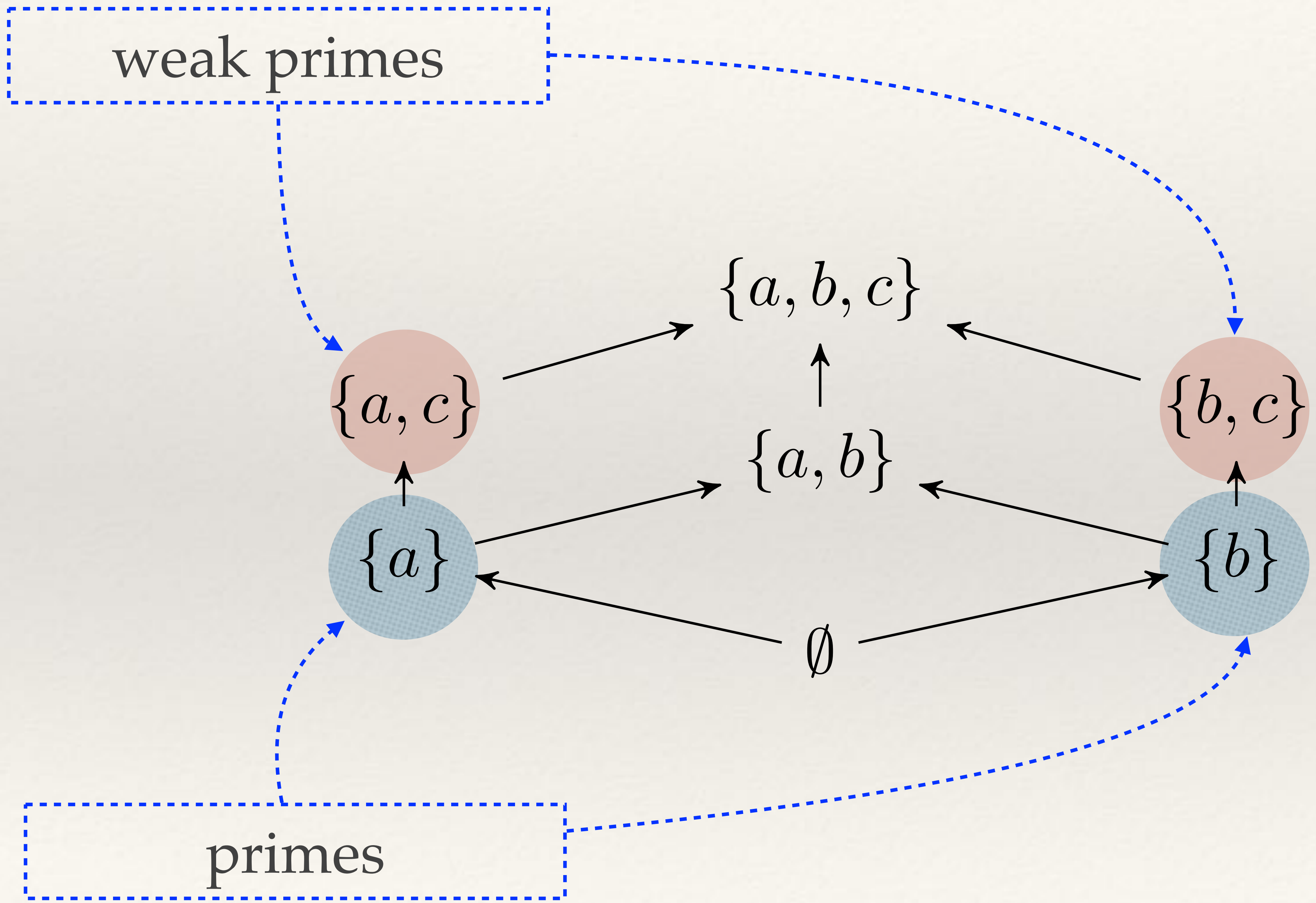
An irreducible *i* of a poset is **weak prime** if

$$i \sqsubseteq \bigsqcup X \text{ then } \exists i'.(i \leftrightarrow i' \text{ and } \exists x \in X. i' \sqsubseteq x)$$

Weak prime partial order

each element is the join of the **weak primes** under it

Weak Prime Partial Orders



Connected ES

Counterpart of prime event structures …

Intuition

ES is **connected** whenever **minimal enablings of the same event are consistent** (transitively)

Formally

$C \overset{e}{\frown} C'$ if $C \vdash_0 e, C' \vdash_0 e,$ and $C \cup C' \cup \{e\}$ consistent

An ES is connected if $C \vdash_0 e$ and $C' \vdash_0 e$ implies $C (\overset{e}{\frown})^* C'$

Example

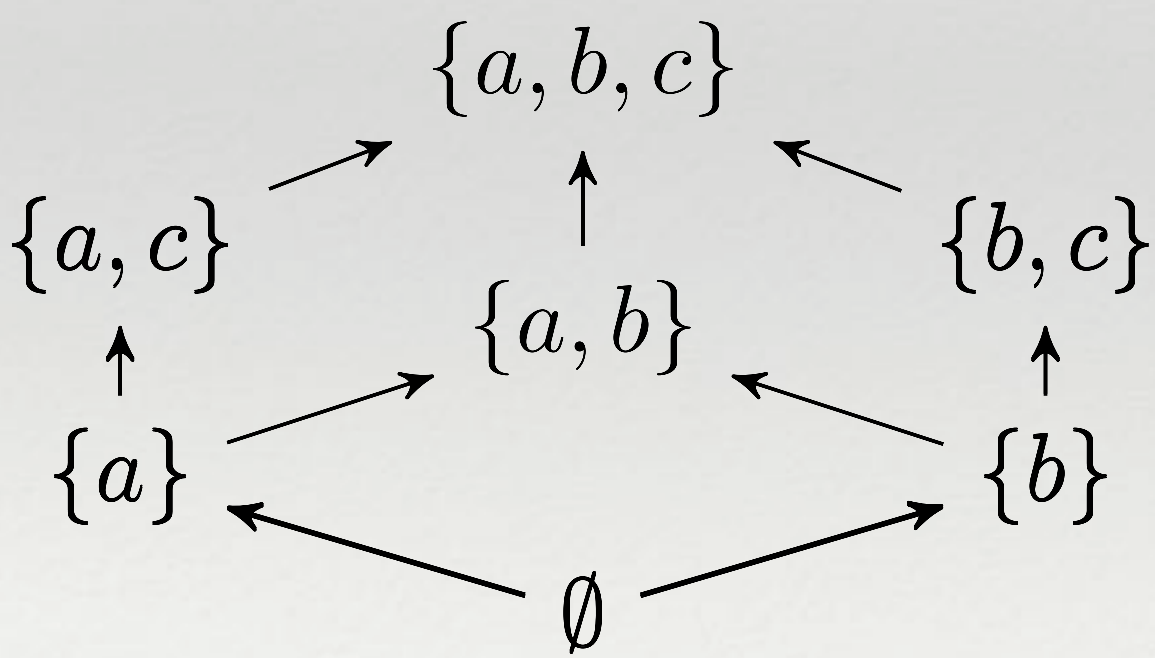
Non-connected

Connected

$\emptyset \vdash a \qquad \{a\} \vdash c$

$\emptyset \vdash b \qquad \{b\} \vdash c$

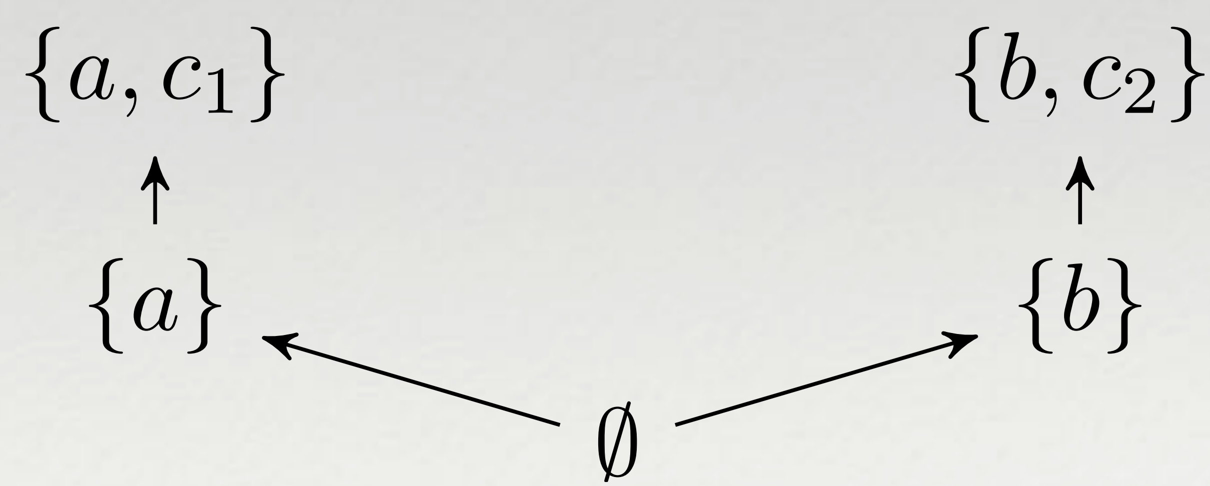
$a \# b$



$\emptyset \vdash a \qquad \{a\} \vdash c_1$

$\emptyset \vdash b \qquad \{b\} \vdash c_2$

$a \# b$



Reference Framework, revisited

configurations
ordered by subset inclusion

Connected Event Structures $\underset{\longleftarrow}{\overset{\longrightarrow}{\cong}}$ **Weak Prime Domains**

weak primes up to $\leftrightarrow$
as events

$\downarrow \uparrow$ traces

Graph Rewriting (with fusion)

Weak Prime Domains vs Graph Rewriting

Graph Rewriting System

G = (start, Rules)

Category of traces **Tr(G)**

- object: **graphs**
- arrows**: traces**, i.e., rewriting seqs up to *shift equivalence*

Then …

- **(start ↓ Tr(G))** is a preorder
- **Idl(start ↓ Tr(G))** is a weak prime domain

Conclusions

Weak prime domains and **connected ES**
(generalising prime domains vs prime ES)

All and only what is needed for **ES semantics** of **graph rewriting with "fusions"**

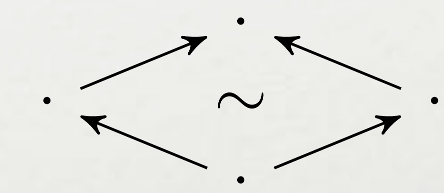
Fusions exactly requires connected (unstable) ES

Forbidding concurrent fusions (injective matching) one recovers a prime ES semantics

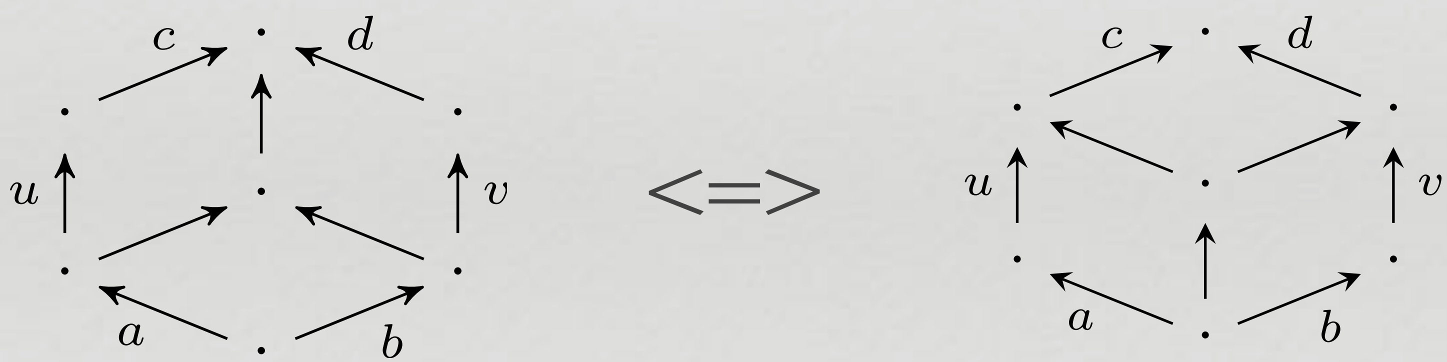
Connected ES ~ prime ES with an equivalence on events

Conclusions

Work on **asynchronous graphs [Melliès]**
LTSs with an equivalence relation on "paths"



PESs = asynchronous graphs where all paths are equivalent and satisfying the **cube property**



Prime algebraicity of the poset of configurations of PESs has a basic role in McMillan-style partial order verification ...

Thanks for listening